ASSOCIATION INTERNATIONALE DE GÉODÉSIE

BUREAU GRAVIMÉTRIQUE INTERNATIONAL

N° 13

Bulletin d'Information

Juin 1966

39 ter, RUE GAY-LUSSAC
PARIS 5e

B U R E A U   G R A V I M E T R I Q U E

I N T E R N A T I O N A L

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B U L L E T I N   D ' I N F O R M A T I O N

Juin 1966

N°13

T A B L E des M A T I E R E S

Bul. n°13

COMMISSION GRAVIMETRIQUE INTERNATIONALE

13-18 Sept. 1965

(Suite du C.R. paru dans les Bul. n°11 et 12)

BIBLIOGRAPHIE

(Les articles indexés sous ces rubriques font suite à ceux mentionnés dans le Bul. (Avril 1962).

VIII - EMPLOI des ANOMALIES de PESANTEUR en GEODESIE

Au cours de la matinée du Jeudi 16 Septembre, le Dr. TENGSTROM, Président du Groupe d'Etudes N°16 de l'A.I.G., "Emploi des anomalies de la pesanteur en Géodésie", fait un exposé sur l'état d'avancement du "travail test dans les Alpes Occidentales", travail qui est destiné à comparer différentes méthodes de calcul pour la détermination gravimétrique des déviations de la verticale, dans une région où la topographie est tourmentée et les isanomales de Bouguer très variables.

Il présente le matériel qu'il a rassemblé et qui doit être utilis par chacun des Groupes de travail. (Voir p. 8).

A la fin de cette réunion, le Prof. MORELLI rappelle que les relevés gravimétriques faits récemment en Méditerranée apportent de nouvelle données pour la carte générale des anomalies.

Puis, quelques remarques sont faites sur le Catalogue des anomalies à l'air libre, établi par K. ARNOLD, en 1964.

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Au début de la matinée du Samedi 18 Septembre et à différentes reprises au cours du Meeting, le Dr. TENGSTROM souligne quelques points importants qui sont résumés ci-après :

1°) - Beside the use of gravity anomalies for the direct solution of the boundary value problem, there are intermediate steps to be considere before the boundary value problem can be attacked. At first, we have to compute the <u>normal heights H</u>. These quantities are not available before we kno the equatorvalue of the model ellipsoid chosen. But this value is not yet known with our terrestrial material alone. Dr. TENGSTROM was of the opinion, however, that it is possible today to compute it on the Potsdam standard to within at least ±1mGal, using information from recent satellitesolutions. The absolute correction to the Potsdam system is not needed for this purpos

An important role for obtaining gravity information to be used for computing correct normal heights is played by the information of Bouguer anomalies and topography. If it is not possible to measure dense gravity along the lines of levelling, the gravity could be interpolated from maps, preferably isanomalymaps of Δg_B and from topographical maps, even if the latter of course are not giving exact H. Using complete Δg_B, the interpolation will be better, but this procedure involves interpolation of terrain-corrections, too, which is tedious and difficult.

But levelling is impossible to carry out in mountainous regions. Here we have to apply trigonometric levelling and transfer the result to potential differences. Dr. TENGSTROM pointed out the importance of Prof. Bodemüller's investigations on this subject. Using gradiometerobservations at the endpoints of a line of sight to obtain more accurate mean values of gravity along it, it is possible to transfer the geometric result to a potential difference, which is needed for the computation of a normal height difference, assuming naturally, that the refraction has been taken into account. Here he mentioned the existence of a Swedish gradiometer for ground or air measurements giving an accuracy in $\frac{\partial g}{\partial h}$ $\pm$ 1 Eötvös. The refraction has been studied simultaneously with vertical angle measurements by interferometric methods and the results seem to be promising. Dr. TENGSTROM wished to encourage all interested geodesists to collaborate with Prof. Bodemüller's Institute in Darmstadt to solve the problem, which also includes the problem of determining instantaneous refraction. He also pointed out the possibility to use interpolation in Bougueranomaly-maps and approximately computed vertical gradients from Rudzki anomalies plus correction for the restored topography to obtain a good picture of mean gravity along the line of sight at trigonometric levelling.

2°) - The importance of gravity anomalies for studying the behaviour of refraction under various weather conditions was also mentioned. He reported about an investigation carried out in Uppsala, where refraction was studied in an area over which the relative geoidal picture was well known from a dense net of free air anomalies. Precise levelling along lines of different azimuts now gives the possibility to compute the value of refraction for each measurement of a vertical angle between the centre and an object, the relative geoidal height and orthometric heightdifference of which is sufficiently well known. The results of these measurements are going to be compared with direct determinations of refraction by means of two interferodispersometers, constructed in Uppsala.

3°) - Gravity anomalies according to Bouguer or Rudzki are also convenient for studying gravity disturbances, caused by buried anomalous masses. Both anomalies have been used together with vertical gradient-measurements and second vertical derivate determinations to calculate shape and depth of old volcanic intrusions in Uppland. The results are very promising.

4°) - At last Dr. TENGSTROM repeated his opinion, that the general solution of the boundary value problem requires also information from vertical gradient measurements (measurements of mean curvatures of level surfaces). We cannot exactly compute vertical gradients unless we have the solution of the Boundary value problem. It is only possible in such a case when we are measuring gravity along a level surface.

5°) - Dr. TENGSTROM regretted it had been impossible for him to quote all important investigations in the field, but he was convinced that all these investigations, from Ohio State University, Institute of Technology in Darmstadt, Osservatorio Geofisico in Trieste, Geophysical Institute in Moscow, Hawaii Institute of Geophysics, etc... were well known and highly appreciated by all geodesists interested in the subject.

Publications distributed at this Meeting :

1) TENGSTROM E. - The present state of the Test Work in the Westalps. Typewritten text, 5 p. + 5 fig. (See p. 7)

- and the material for this work.

2) BRAGARD L. - Methods to determine the shape of the topographical Earth surface by successive regional determinations of this shape in function of regional gravity measurements or in function of these and regional vertical gravity gradient measurements.
Typewritten text, 12 p.

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COMPLEMENTARY INFORMATION

Papers presented to the Sp. St. Gr. N°16 (Sec. V of the I.A.G.), Uppsala, Sweden. (July 1965).

1) ARNOLD K. - Uber die Berechnung des gravimetrischen Korrektionsgliedes.

2) BRAGARD L. - Methods to determine the shape of the topographical Earth surface ...
(see above n°2)

3) BURSA M. - Molodenskij's Formulas and their Application to the Test-Area work.

4) CARROZZO M.T. & C.MORELLI - On the Definition of Bouguer Anomalies referred to Model Earth.

5) FISCHER I. - Gravimetric Interpolation of Deflections of the Vertical by Electronic Computer.

6) GRAAFF-HUNTER J. De - The difference between the Model Earth M.E. and the Actual Earth A.E.

7) LEDERSTEGER K. - The Equilibrium Figure and the Normal Spheroid of the Earth.

8) MELCHIOR P. & E. TENGSTROM - La station de marées terrestres de Dannemora.

9) PICK M. - The Determination of the Axis of Rotation Inclination of the Reference Ellipsoid towards the Earth's Axis of Rotation.

These papers are being published in Medd. från Geod. Inst. Uppsala. N°9.

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Recent book :

AMERICAN GEOPHYSICAL UNION - Gravity Anomalies : Unsurveyed Areas - Pub. Nat. Acad. Sci. March 1966. Geophysical Monograph 9. 142 p. Washington.

- Existing Surface Gravity Material (UOTILA U.A.)
- The Extension of the Gravity Net to the Unsurveyed Areas of the Earth : Statistical Methods (RAPP R.H.)
- The Extension of the Gravity Net to the Unsurveyed Areas of the Earth : Geophysical Methods (HEISKANEN W.A.)
- External Gravity Field of the Earth (MUELLER I.I.)

ANNEXE I

THE PRESENT STATE OF THE TEST WORK IN THE WESTALPS

by Erik TENGSTROM

Introduction

This work, which is being performed to test numerically and in rugged topography with complicated Bouguer-structures, several methods for accurate gravimetric determination of deflections and deflectiondifferences was proposed during the IGC-meeting in Paris 1962 by Prof. MORELLI.

The working group on "The use of gravity anomalies in Geodesy", created at the same meeting, began the preparation for defining precisely the aim of the study and for collecting necessary gravimetric and astro-geodetic material. Conferences of this working-group in Trieste 1963 and of SSG 16 - to which the work has now been transferred - in Paris 1964 stated the main lines of the investigation and I took myself the responsibility for assembling the data which should be used by all representatives for different methods. These data are now on hand and will be delivered to five such representatives during the meeting here. The data will be described below. In Prague 1964 during the Symposium on "The determination of the figure of the Earth" it was agreed, that I should conduct the work and deliver the material in such a form as I had already proposed in Trieste. At a conference of SSG 16 in Uppsala July 1965 the first copies of the data were distributed to four investigationgroups. This material which suffered from several computational and transcription errors and lacked a detailed analytical description of the Bouguerfield in the inner zone (see below) has now been rechecked by myself and an interpolationformula for a grid of 5' x 5' accepted point Bouguervalues has been calculated for each square 5' x 5'. Five copies of this interpolationformula are available here in Pari

The center of the investigation area is 45° N latitude, 8°33' E longitude. Inside a circle of radius 100 km it contains 20 astronomical stations with latitude - and longitude determinations or latitude - and azimut observations. The deflectioncomponents are calculated in ED and have an average internal mean error of ± 0".20 for ξ and ± 0".15 for η from longitude, ± 0".30 or bigger for η from azimuth. The distances between the astro-stations vary from 8 km to 190 km.

Gravity and topo-material

The necessary gravimetric material, to be used for the different test-groups will be given in the following form :

Area 1 : Catalogue of all available complete Bougueranomalies with positions, heights and terraincorrections (2.67,0_2) from the center out to approximately 200 km. (See also fig.1). Number of anomalies in catalogue about 2400. Mean distance between the stations ~ 8 km. The terrain corrections for the Swiss stations are, however, computed out to max 50 km and the topographic corrections for some Italian stations have been interpolated. Using this material, the statistical method of Dr. RAPP, Ohio State University, has been utilized in constructing a grid of most probable complete Bougueranomalies for 5' x 5' (fig.2)

This work has been done under the supervision of Prof. UOTILA, ohio State University. Regarding the covariance to be big for the grid-distance, we have thought it advisable for the first deflection-computation in the testwork to accept the gridnumbers as giving the correct values of the Bougueranomalies. Therefore, to study the low derivatives in the gravimetric field, an orthogonal expansion of absolute norm in polynomials of 10^{th} degree around the center of each gridsquare has been performed. The result in 5 copies will be delivered to active groups during this meeting. An analytical interpolation with orthogonal polynomials of 16^{th} degree using the same material has also been made for comparison. For all methods a detailed information of topography is also necessary in this area. For that reason the following topographical maps are delivered to all groups :

a) - 1 : 25 000 Italian map without numbered grid, covering a circle of 10 km radius around each astrostation. Equidistance of heightcurves 5-10 m.

b) - 1 : 200 000 Italian map, covering a circle of ~ 100 km around each astrostation. Equidistance of curves 100 m.

In addition, bathymetric maps of the bigger lakes in the area are available (1 : 50 000) and a bathymetric map of the Ligurian Sea (1 : 750 000).

Area 2 : The material consists of mean complete Bougueranomalies for squares of 20' lat x 30' long with mean elevation and mean terraincorrection, (2.67,0_2). See fig. 3, which also indicates the different sources used and an estimated expected standard error for free-air anomalies, calculated from this material. The terraincorrections have been meaned in the squares, where this was possible, in other squares we have used the following approximate method : for landsquares a correlationcurve mean height-mean terraincorrection has been constructed from squares with known mean values. This curve shows a standard deviation of ~ $\pm$ 3 mGal. Against each mean height the value of topographical correction has been taken and plotted.

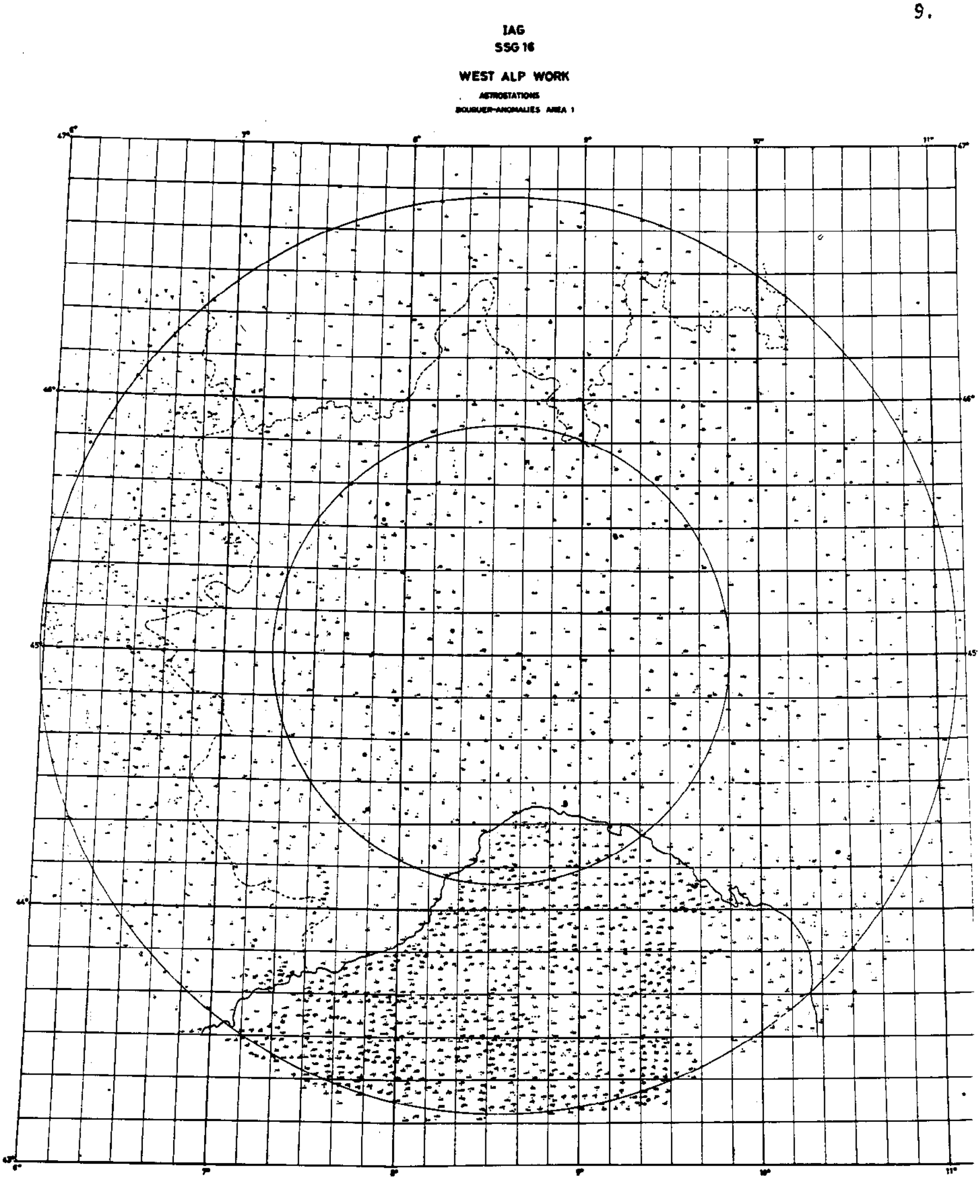

IAG
SSG 16
WEST ALP WORK
ASTROSTATIONS
BOUGUER-ANOMALIES AREA 1
6°
7°
8°
9°
10°
11°
47°
46°
45°
44°
43°

Fig. 1

IAG
SSG 16

WEST ALP WORK

ASTROSTATIONS
INTERPOLATED BOUGUER-ANOMALIES AREA 1

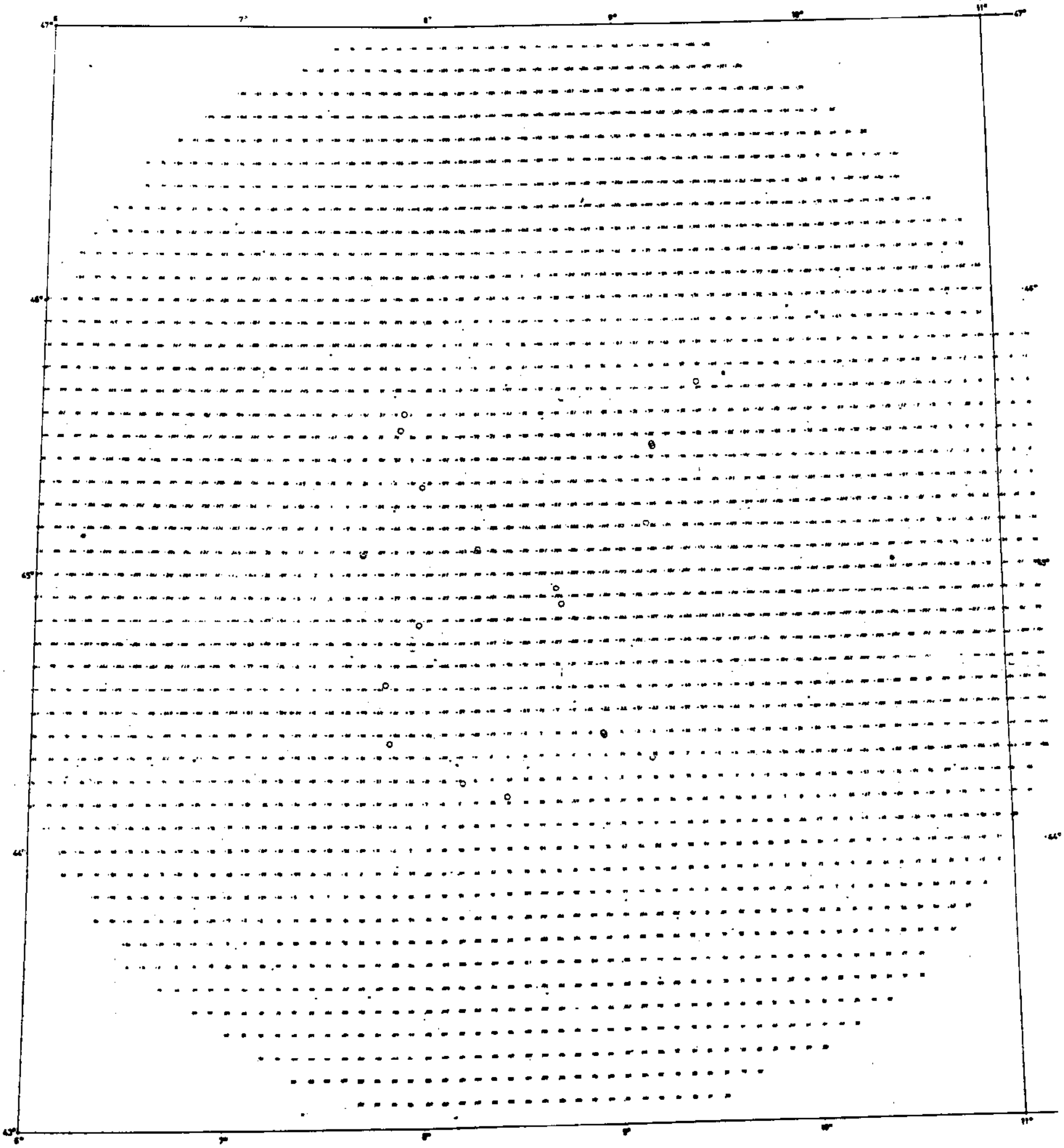


Fig. 2

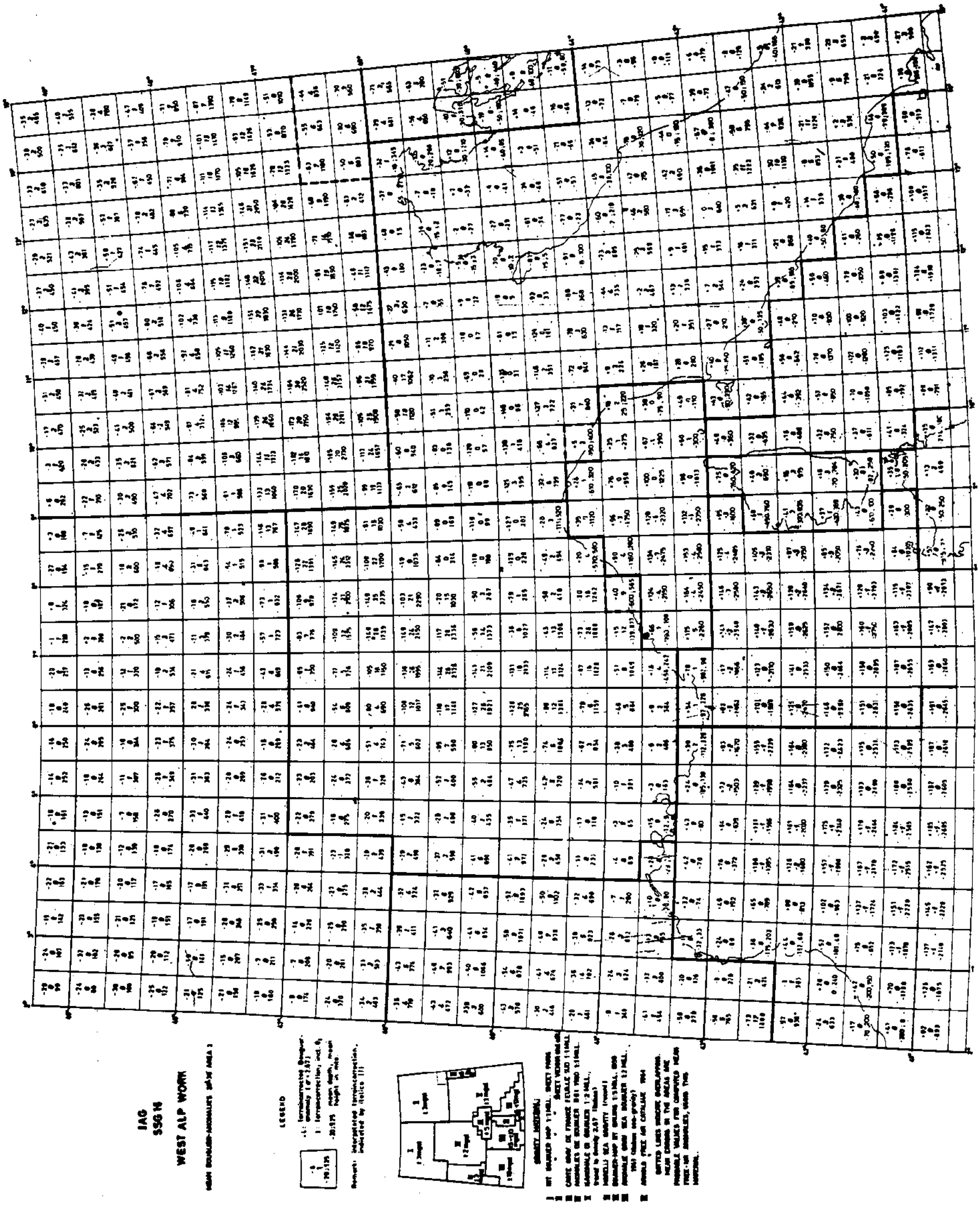


Fig. 3

For seasquares, the mean terraincorrection has been evaluated from the contribution of 8 surrounding squares, regarded as having the depths equal to their mean depth and from the corresponding graphically interpolated contribution of each 1° x 1° degree square of 8 such squares, using their mean depths. A correction for the effect of greater distances in the second case and for a probable topographical curvature in the 20' x 30' square for the first case has been applied. The second correctionfactor has considered three cases :

1) - The square has constant curvature (open sea).
2&3) - Bigger curvatures, the bigger the more the coastlines surround the square in question. For combined sea and landsquares the mean of both methods has been taken.

Area 3 : The material is given as mean free-air anomalies and mean terraincorrections for squares of 1° x 1° covering a circle of approximately 1 600 km radius round the conter of the testarea. The result is shown in fig. 4 with estimated standard error, and the gravimetric and topographic sources are indicated in fig. 5.

Despite its minor importance for the originally outlined testwork, this map (fig. 4) has a very great value for the calculation of an absolute deflection and geoidal height at the center of the testarea. It has been agreed upon (Uppsala 1965), that such a calculation should be made by the Geodetic Institute in Uppsala and the result be given together with the report of its computation of relative gravimetrical deflections at the given astro-stations. The work to construct this map has been very laborious and contained big difficulties due to the inhomogenous material in Bougueranomalies and elevations. The indicated average mean errors should, however, be sufficiently representative for an estimation of the accuracy in the contribution from this area to ξ , η and N. The terraincorrections are evaluated for landsquares by means of a correlationcurve mean height - mean terraincorrection for 1° x 1° squares, constructed from known values, (France, Switzerland, Italy). The standard deviation from the curve is $\sim \pm$ 2 mGal. For sea-squares and combined squares the same principles as mentioned above for area 2 have been followed.

Area 4 : 1 620 km from the center until the antipode. Here mean free-air anomalies of all available 1° x 1° and 5° x 5° squares have been used together with 40° x 40° means from recent satelliteresults. The mean free-air anomalies from the satellite-solutions do not show any systematical error in the equatorvalue of gravity, used in the international gravity formula, and this statement was concluded from a comparison between "measured" 40° x 40° means and the corresponding satellite-results. Instead of giving the free-air field for this big area, a computation of its contribution to ξ , η and N has been performed at the Geodetic Institute in Uppsala.

The result of this computation is :

$$\Delta\xi = -0''.94$$
$$\Delta\eta = -0''.19$$
$$\Delta N = +11 \text{ m}$$

From several considerations as regards the accuracy of the used material, we got a rough estimation of the mean error in the result from the global field. This is in ξ and η less than $\pm$ 0".2, in N $\pm$ 3 m.

Methods to be used for the calculation of deflectiondifferences in the testarea.

I . Molodenskij-method, represented by the Molodenskij-group in USSR and by Dr. BURSA and Dr. PICK in Prague.

II. Method by Arnold K., represented by the Potsdamgroup.

III. Method by Bjerhammar A., represented by a group at the Department of Geodesy, Institute of Technology, Stockholm.

IV. Model Earth method by Dr. de Graaff Hunter J. , represented by Prof. MORELLI and Brig. GLENNIE

V . Rudzki-reduction method, represented by a group at the Institute of Geodesy, Uppsala.

These five groups intend to start their computations - or have already started them - so that the result can be reported at the General Assembly of IAG in 1967.

The values of the astrogeodetic deflections in ED are in part on hand, but cannot yet be delivered in full before the η-components from azimuthobservations have been checked once more.

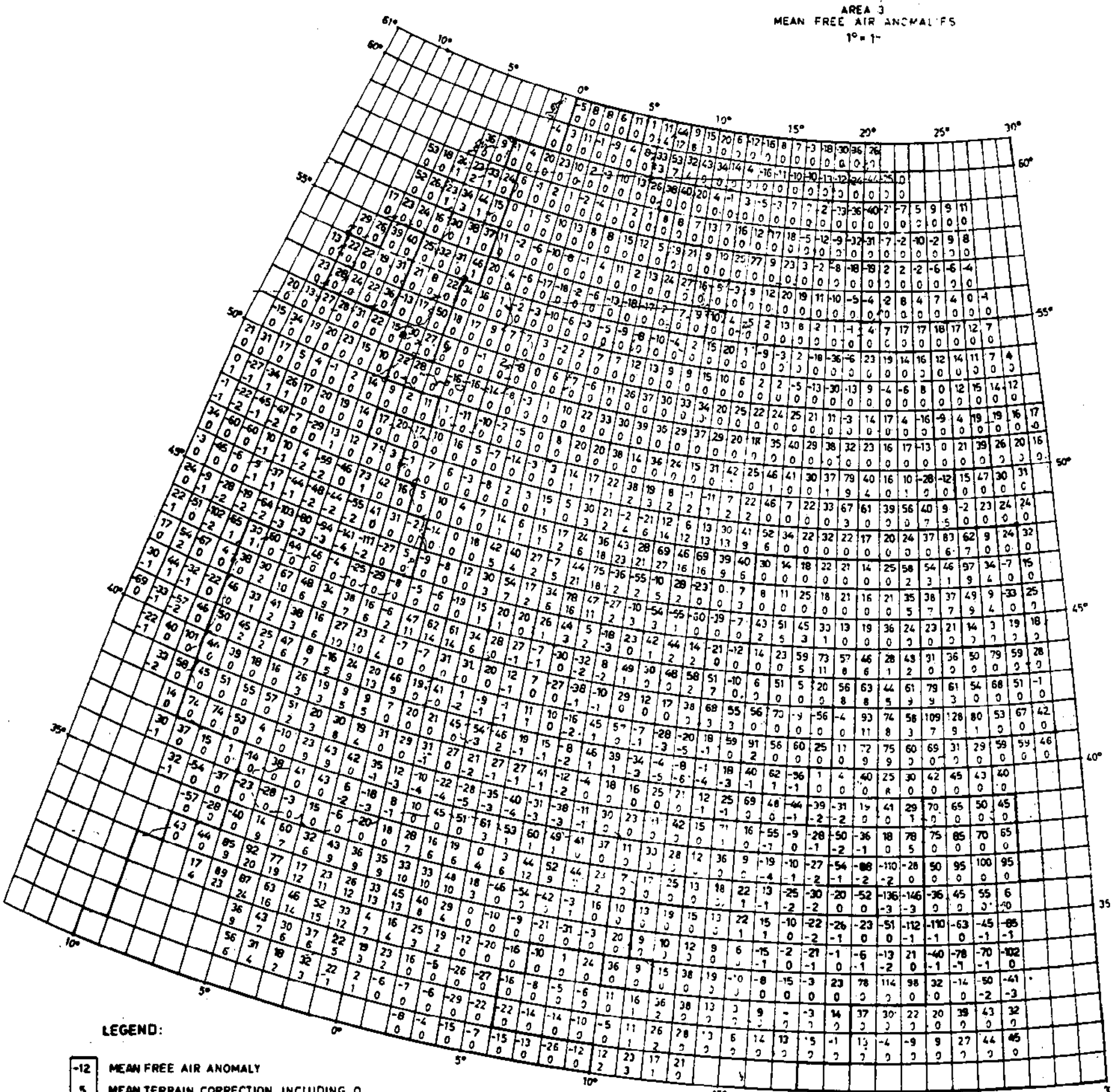
A detailed description of abovementioned methods for deriving surface-deflections will be attached to the "comptes rendus" of the SSG 16 conference in Uppsala, July 1965, which are going to be published during this fall.

The abovementioned testwork is now in its first stage, only. An improvement of all material by means of new measurements and refined treatment of measured data is intended to be made for the next step. For this second investigation I would like to suggest the use of all kinds of reliable interpolationmethods for the gravity material, also such methods pendin on geological and geophysical information, as proposed by Prof. WOOLLARD.

IAG
SSG 16

WEST ALP WORK

AREA 3
MEAN FREE AIR ANOMALIES
1° × 1°



LEGEND:

-12	MEAN FREE AIR ANOMALY
5	MEAN TERRAIN CORRECTION, INCLUDING Q_0

SQUARES INSIDE HEAVY CONTOUR MEAN ERROR ± 5 mgal
OTHER SQUARES ± 10 mgal

Fig. 4

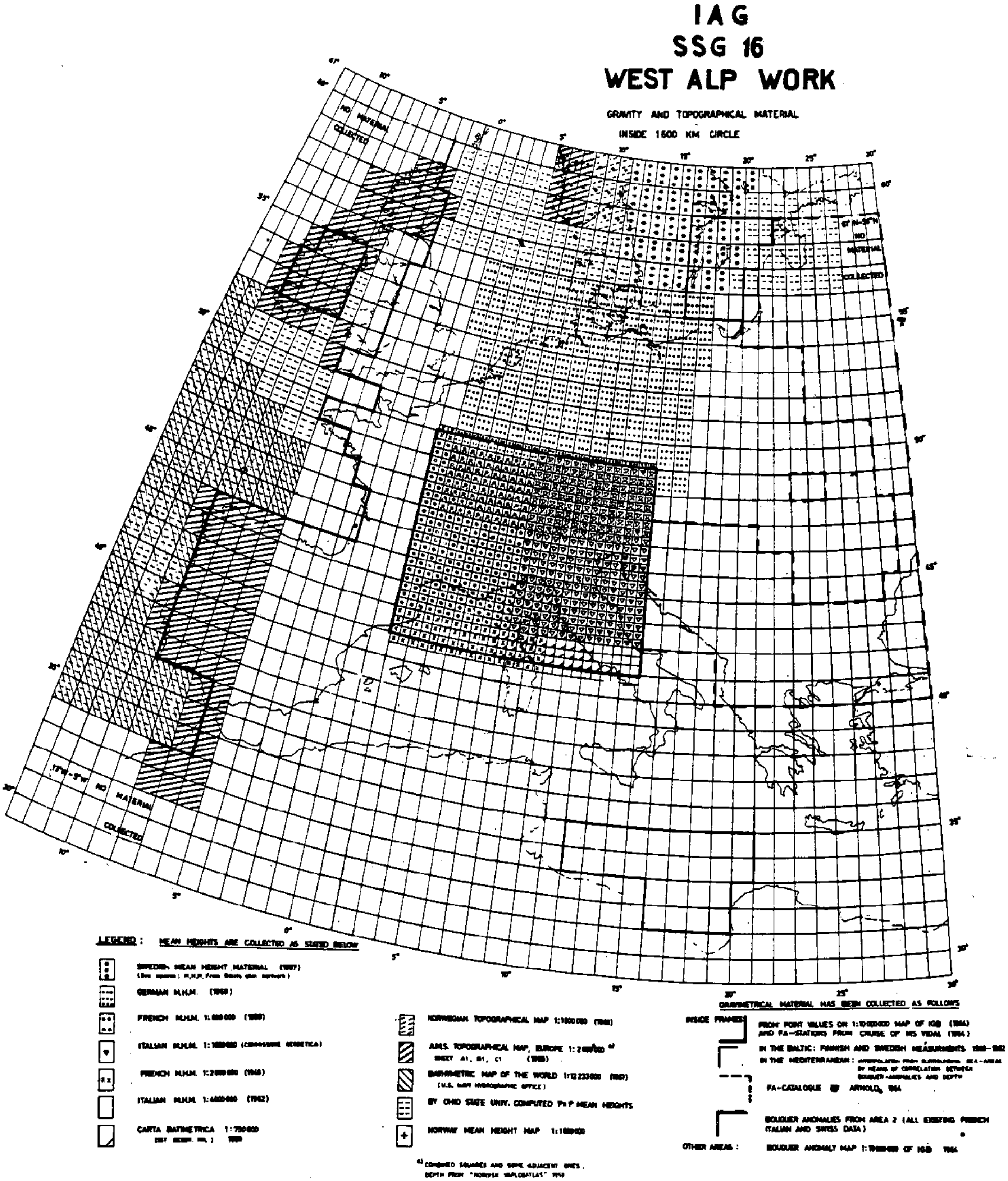

IAG
SSG 16
WEST ALP WORK
GRAVITY AND TOPOGRAPHICAL MATERIAL
INSIDE 1600 KM CIRCLE
NO MATERIAL COLLECTED
NO MATERIAL COLLECTED
NO MATERIAL COLLECTED
LEGEND: MEAN HEIGHTS ARE COLLECTED AS STATED BELOW
GERMAN M.H.M.
FRENCH M.H.M.
ITALIAN M.H.M.
FRENCH M.H.M.
ITALIAN M.H.M.
CARTA BATIMETRICA 1:750000
NORWEGIAN TOPOGRAPHICAL MAP
AMS. TOPOGRAPHICAL MAP, EUROPE
BATHYMETRIC MAP OF THE WORLD
BY OHIO STATE UNIV. COMPUTED MEAN HEIGHTS
NORWAY MEAN HEIGHT MAP
GRAVIMETRICAL MATERIAL HAS BEEN COLLECTED AS FOLLOWS
INSIDE FRAMES
IN THE BALTIC: FINNISH AND SWEDISH MEASUREMENTS
IN THE MEDITERRANEAN:
FA-CATALOGUE
BOUGUER ANOMALIES FROM AREA 2 (ALL EXISTING FRENCH ITALIAN AND SWISS DATA)
OTHER AREAS:
BOUGUER ANOMALY MAP OF IGB

Fig. 5

IX - GRADIENT VERTICAL de PESANTEUR

Prof. CONSTANTINESCU presents his Report (1) which was distributed to the delegates at the end of the meeting.

This report is a very general review of the problem concerning the vertical gradient of gravity. A list of bibliographic references is added at the end.

He emphasizes the great interest presented by the anomalous values of the vertical gradient comparatively with the normal values, for both geodetical and geophysical research and applications. In particular he recalls the various ideas about the use of the vertical gradient in the free air correction for the reduction of the observed gravity : ones think the normal gradient is to be used, the others the anomalous values, taking account of the local or regional variations. He also points out the problem of the vertical gradient in connexion with studies concerning the determination of the geoid.

Then, he summarises the three ways for determining the vertical gradient :

a) Attempt of direct measurement,
b) Indirect measurement by means of "vertical measurements of gravity",
c) Derivation by computation from horizontal plane distributions of gravity values.

Finally, Prof. CONSTANTINESCU gives some information on the uses of the vertical gradient of gravity. The actual value has been taken into account not only for correcting gravity data but also for describing more accurately the processes involved in absolute determinations. However, the most important application remains in the field of geological gravimetry, in the interpretation of gravity anomalies. (Study of special geological features, determination of the rock densities, etc...).

Dr. TENGSTROM referred to the role of the vertical gradient of gravity (and higher vertical derivatives) which will help to find the solution of the boundary value problem of Geodesy. His opinion was that these quantities should be measured together with the gravity method to ascertain a correct solution which will be used in the case of a very rugged topography and Bouguer field.

In the case of Stokes (oceanic Earth), where the boundary problem is uniquely solveable with respect to the Equipotential Model from correctly computed free air anomalies, he preferred the use of a formula for the vercial gradient (mean curvature of the geoid), which is easily derived from Stokes'formula. This formula could also be used for mean curvatures at the surface of Earth models with small slopes, but we have always to be aware of its character of being an approximation beyond the approximation accepted by Stokes.

Dr. BARANOV insiste sur le fait que la formule appelée intégrale de Poisson, pour le plan, peut aussi être étendue à la sphère ; dans ce cas, on obtient une formule pour le gradient vertical qui est valable en "géodésie sphérique". Mais la principale difficulté consiste dans le fait que g est connu sur la surface topographique irrégulière : il se pose donc un problème de réduction à une sphère ou à un plan ; pratiquement, ce problème a été assez bien résolu par NAUDY H. & R. NEUMANN (3).

Dr. PELLINEN mentioned that "the plane of spherical formulae for the calculation of the vertical gradient may give wrong results if gravity anomalies are referred to the physical surface. It is seen from the new model investigations (Trudy Cniigaik, Moscou, 1955, v.157) that the errors may be about 20 % of the gradient value and greater".

Prof. MORELLI recalls the work of Prof. KUMAGAI and points out the importance of the local values of the vertical gradient of g.

Publications distributed during the Meeting :

(1) CONSTANTINESCU L. - Main trends in research and applications concerning the vertical gradient of gravity -. Typewritten text, 14 p. (See abstract p.19)

(2) CONSTANTINESCU L. & M.M. ABD ELDAIEM - Une formule pratique pour l'approximation du gradient vertical -. (Texte roumain, résumés russe et français). Probl. Geof. (Acad. R. Pop. Rom.), 1963, v.II, p.27-44.

(3) NAUDY H. & R. NEUMANN - Sur la définition de l'anomalie de Bouguer et ses conséquences pratiques. Geoph. Prosp. 1963, v.13, n°1.

To be added, the following publication not yet mentioned in the last Bul. Inf.

(4) GERMAN S. - Die Waage im inhomogenen Schwerefeld -. Z. Feinwerktechnik, 1960, v.64, n°10, (DK 526 77) s.1-5.

Some results of vertical gradient measurements are given in the table 1.

Note : Dr. THOMPSON Llyod G.D. (who could not attend the Meeting of the I.G.C.) sent at a later date the following paper :

THOMPSON L.G.D. & W.B. AGOCS - Techniques and applications of gravity Gradient measurements. (Geo Space Gravity Observatory).

"The recent development of a prototype gravity gradiometer and high sensitivity accelerometers has stimulated renewed interest in gravity gradient techniques and applications. Programs for the development and use of vertical and horizontal gradiometers for geophysical and geodetic applications, and to make gravity gradient measurements on orbiting spacecraft to determine the gravity fields of the earth and moon are in progress."

ANNEXE II

MAIN TRENDS in RESEARCH and APPLICATIONS CONCERNING the VERTICAL GRADIENT of GRAVITY

by Liviu CONSTANTINESCU

(Extract) *

...

THE VERTICAL GRADIENT OF GRAVITY IN GEOPHYSICAL CONTEXT

As many other problems concerning the gravity field, the vertical gradient of gravity ($g_z = V_{zz}$) has geodetical as well as geophysical implications. First of all, it plays an important part in reducing or correcting the observed values of gravity in order to make them comparable and consequently interests both, physical geodesy (or geodetical gravimetry) and gravity prospecting (or geological gravimetry).

...

In the second place, the problem of the vertical gradient of gravity has a well-known bearing on geodesy in connexion with studies concerning the determination of the geoid. The formulae recommanded for obtaining this surface as defining the figure of the Earth (Molodenskij, 1958) involve for its application the use of the anomalies of the vertical gradient of gravity. Recently, new possibilities have been shown for computing the two first vertical derivatives of gravity with a consistent accuracy with such geodetical requirements. (Jurkina, 1965). Another geodetical implication of the problem of the vertical gradient is connected with the possibility, indicated by means of theoretical considerations, of "deducing the principal curvatures of points of equipotential surfaces and of azimuth of the lines of curvatures only from gravity and from measured vertical gradients of gravity" (Bodemüller, 1963).

* The integral text will be published in : "Revue de Géologie et de Géographie". Academia Republicii Populare Romine, Bucarest. t.9, n°2.

The third important point of interest presented by the vertical gradient of gravity is its capacity of closely contouring by the isolines of its spatial distribution, the location of the anomalous masses constituting the sources of the gravity anomalies.

...

DETERMINATION OF THE VERTICAL GRADIENT OF GRAVITY

The attempts at measuring directly the vertical gradient of gravity have remained till now without success. Although the theory of such a measurement has been carefully elaborated, technical difficulties in giving reality to its instrumental equivalent have not yet been overcome. (Hammer, 1963). Optimism in this respect seems nevertheless justified, (Chinnery, 1961 - Paterson, 1961), since that the corresponding problem for the geomagnetic field has been solved very satisfactorily.

The so-called vertical measurements of gravity (Domzalski, 1955), serving as a basis for the "experimental determination" of the vertical gradient of gravity (Balavadze & Shengelaya, 1954), consist in carrying out gravity measurements at two different heights on the same vertical. The vertical gradient is then measured by the ratio of their differences and the distance separating the two measurement points. Though severely limited by the condition of associating a surface measurement of gravity with an above surface (or under surface), this way of attacking the problem of determining the vertical gradient has led to satisfactory results, for both geodesy (Bodemüller, 1963) and gravity prospecting. (Balavadze, 1955)

Already applied in underground measurements (Domzalski, 1955 - Thyssen-Bornemisza, 1964), the principle of this procedure is liable to be conveniently adapted to the conditions of "airborne gravity gradient measurements". (Paterson, 1961).

The third way of arriving at a quantitative knowledge of the vertical gradient of gravity (or nearly so), viz. the computation of this quantity on the basis of the horizontal plane distribution of gravity values, has been mostly trodden, various possibilities offered in this respect by the potential theory and its consequences having been widely exploited. Three main types of procedures are to be mentioned as most representative :

a) Differentiation of the gravity as expressed by the Poisson formula,

b) Surface integration of the second derivative of gravity,

c) Comparison of a downward and upward analytical continuation of the gravity values as known in the datum plane.

The first way may be considered as the oldest successful attempt at computing the vertical gradient of gravity (Evjen, 1936) and has led subsequently to numerous and various practical procedures and approximating formulae (Baranov, 1953 - Veselov, 1954 - Balavadze, 1955 - Ochapovski & al., 1956 - Tiapkin, 1957 - Kantas, 1959). Even when the connexion of these practical procedures with the initial idea of Evjen is less obvious they may be reduced, in the last resort, to the same basis principle. From the practical point of view, they are all characterised by the same approach to exploit the gravity data as known by their horizontal plane distribution: multiplication of gravity mean values on circles by numerical coefficients depending on the form and size of the geometrical configuration of points used for collecting the observational data and summation of the terms thus obtained. As to their differential marks, they may manifest themselves in the mentioned configuration (grid, spacing, point distribution), its lateral extension, the use of templates, the degree of standardization, etc... whence the variable practicality of the procedures and the differences in the accuracy of the results they lead to.

Strange as it may appear, the idea of computing the vertical gradient with the aid of the horizontal plane distribution of the values of the second derivatives of gravity is suggested by the solution of the Neumann problem of the potential theory. It has been shaped by Ackerman & Dix (1955), into a practical procedure consisting in a surface integration carried out on a second derivative map, previously deduced from a gravity map. Without claiming anything new in theory, this procedure has proved to be convenient in special practical cases.

The third mentioned possibility of obtaining the vertical gradient of gravity is provided by computing it in terms of two analytical continuations of the gravity, carried out at the same vertical distance from the horizontal datum plane, one downwards and the other one upwards. Dividing their difference by twice the distance of continuation gives a quotient representing an approximation of the vertical gradient. A practical formula may be obtained by combining into a single operation the two continuations as well as the division of the difference between the gravity values thus arrived at, and the corresponding vertical distance. Under special conditions concerning the point configuration and the height, respectively depth of continuation, the approximation may be quite satisfactory (Constantinescu & Eldaien, 1963).

To these three types of procedures permitting the computation of the vertical gradient of gravity one may add some others, differing more or less from them by both the basis principle and manner of application. Such as might be quoted, e.g., the geodetic method of Muller (1961) allowing to compute the vertical gradient of gravity by means of the components of the deflection of the vertical as well as that of Jurkina (1965), both interesting for geodesy, in studies of the form of the equipotential surfaces, (in particularly the geoid.)

For both geodesy and applied geophysics is worth mentioning the computing procedure of Morelli & Carrozzo (1964), based on the knowledge of gravity anomalies as provided by gravity meter measurements. On the other hand, an interesting, though cumbersome method given by Haalck (1950) offers the possibility of calculating the vertical gradient of gravity (as well as other gravimetric quantities) from observational results provided by the torsion balance. Finally, the computation of vertical gradient values appears as an important by-product of the application of the relaxation method to upward continuation of gravity data (Roy & Burman, 1960).

On the whole, one might characterize the present state of the problem of determining the vertical gradient of gravity by the lack of a direct measurement apparatus and the existence of indirect methods. The latter ones consist either in approximating the vertical gradient by the quotient whose limit defines it (the quantities in the quotient being directly measured) or in deducing this gravimetric element by computation from the surface distribution of the gravity values, within the framework of some theorems of the potential theory.

APPLICATIONS OF THE VERTICAL GRADIENT OF GRAVITY

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CONCLUSIONS

Presenting interest for both physical geodesy and applied geophysics in connexion with the correction of gravity data as well as for the intrinsec geodetical or geophysical study of gravity anomalies, the vertical gradient of gravity cannot yet been directly measured. A theory of its metrology is already elaborated and a clear tendency of giving it reality is to be recognized in efforts due to overcome the existing technical difficulties.

Two ways for obtaining quantitative (or, at least, quasi-quantitative) information on the vertical gradient of gravity are now available : experimental determination by "vertical measurements of gravity" and derivation by computation from horizontal plane distribution of gravity values. Efforts are being made for improvements in both directions, the first leading to local values, the second one providing rather regional values of the vertical gradient of gravity.

From the point of view of the applications, the preference is to be quoted for using regional anomalous gradients for gravity corrections and for geodetical research problems - as they seem to be more suitable for such purposes -, while in connexion with the local values of this quantity the trend is apparent to apply them prevailingly in practical problems of gravity prospecting.

In spite of its shortcomings - in particular its sensitivity to the influence of lateral masses and, ipso facto, to the irregularities of the topographic surface -, the anomalous vertical gradient has proved to be quite useful in the field of applied geophysics for problems like the closer contouring of gravity anomalies, their resolution in cases of cumulative effects and, under favourable conditions, for quantitative analyses. The great interest thus presented by the vertical gradient explains another conspicuous trend in its metrology : attempts at introducing airborne measurements in this field too.

ANNEXE III

DETERMINATION DE LA FIGURE, DES DIMENSIONS ET DU CHAMP GRAVIFIQUE DE LA TERRE

(Extrait du Rapport de l'Académie des Sciences U.R.S.S. Comité Soviétique de Géophysique)

by

Ju. D. BOULANGER & L.P. PELLINEN

Dans toute une série de travaux V.V. BROVAR a décrit ses recherches concernant de nouvelles solutions du problème de Molodenskij sur la détermination du champ de pesanteur à l'extérieur de la Terre, d'après les valeurs de la pesanteur à la surface.

L'essentiel de ces recherches est donné brièvement par V.V. BROVAR dans son rapport établi à l'occasion de la treizième Assemblée Générale de l'U.G.G.I. et publié dans le Bulletin Géodésique, (1964, n°72, p.167-173). Les travaux sur l'estimation et l'étude de ces méthodes sont poursuivis. A la base de ses formules pour le potentiel perturbateur et ses dérivées hors de la Terre, V.V. BROVAR a considéré la solution de l'équation pour la densité d'une simple couche.

La déviation de la verticale à la surface physique de la Terre est exprimée par les valeurs de l'anomalie de la pesanteur et du potentiel perturbateur aux points de cette surface. Dans les expressions du potentiel perturbateur, de la déviation de la verticale et des constantes de Stokes, basées sur l'équation intégrale de Molodenskij, on a mis en relief les approximations de Stokes. On a analysé l'expression de la déviation de la verticale pour un modèle de Terre ayant la forme d'un cône situé sur le plan de référence. Ce principe des calculs à la base de l'équation intégrale de Molodenskij pour le potentiel perturbateur, a été étudié pour le même modèle par JURKINA & STAROSTINA. Dans leurs articles, JURKINA & ALEKSACHINA ont considéré quelques termes correctifs pour l'expression de la constante de Stokes ainsi que la valeur moyenne du carré des pentes en terrain montagneux.

L.P. PELLINEN a proposé des méthodes qui simplifient les calculs des caractéristiques du champ de pesanteur terrestre d'après les formules de Molodenskij, en première approximation.

V.F. EREMEEV a précisé les formules concernant les altitudes "normales" et a donné un aperçu des travaux des auteurs occidentaux concernant les calculs des altitudes "normales et des autres. Il a noté des erreurs dans la détermination de l'altitude "normale" ainsi que donne la critique des ouvrages : A.A. RUNE & J. VIGNAL, O. SIMONSEN, J. BOKUN, T. CHOJNICKI et les autres.

Dans ses articles V.I. ARONOV a décrit un procédé de calcul électronique et calcule aussi sur des modèles de Terre, les éléments du champ de pseanteur à l'extérieur de la Terre (déviations de la verticale et anomalies de pesanteur) à l'aide des valeurs Δg sur le plan de référence. Un aperçu est donné des méthodes de réduction sur un plan, des anomalies de pesanteur observées à la surface terrestre. V.I. ARONOV propose de calculer les anomalies de pesanteur sur le plan inférieur à l'aide de l'intégrale de Poisson en considérant celle-ci comme une équation intégrale du premier ordre. Le pas d'intégration doit être à peu près égal à celui du levé. Comme il est impossible dans le cas général d'expliquer le champ des anomalies de la pesanteur à la surface physique de la Terre par la présence du champ sur le plan inférieur, la solution du problème peut-tre imprécise. V.N. STRAHOV a décrit un procédé de calcul (avec un calculateur électronique rapide) du gradient vertical de pesanteur, à partir des anomalies données sur un plan. Il a étudié également les procédés de calculs pour la détermination des dérivées verticales supérieures. M.J. JURKINA a exprimé le gradient vertical à partir des anomalies à la surface physique de la Terre.

M.J. MARIOTH a décrit le principe de la détermination du géoide régularisé en tenant compte des valeurs d'ordre n, sans utiliser le champ normal. La distance ξ entre la surface de référence et la surface déterminée doit être une petite valeur du même ordre, c'est à dire d'ordre $\xi \alpha^{n-2}$ où α est l'aplatissement de la Terre. Dans cette méthode, on n'a pas besoin de déterminer les anomalies du gradient vertical de pesanteur ni de résoudre le problème de Stokes dans le cas d'une surface autre que celle d'une sphère ou d'un ellipsoide. Dans les calculs à partir de $n = 3$, on utilise les altitudes du géoide et les déviations de la verticale supposées déterminées dans l'approximation précédente. Pour $n = 3$, la méthode proposée est étudiée en détail et appliquée à un modèle de la Terre.

J.F. MONIN a étudié les méthodes de la détermination du géoide régularisé avec une erreur relative de l'ordre du carré de l'aplatissement de la Terre. Il a également examiné les méthodes de détermination du champ gravifique extérieur (en se basant sur l'équation intégro-différentielle pour le potentiel perturbateur) et son application sans utiliser le champ normal.

Dans ses ouvrages, quelques erreurs ont été relevées, soit par l'auteur lui-même, dans ses articles postérieurs, soit par les auteurs suivants : N.K. MIGAL, V.F. EREMEEV, et M.J. JURKINA.

Selon les calculs de N.P. GROUCHINSKI et M.O. SAGITOV, l'ignorance des courants marins lors des mesures de gravité en mer, peut entraîner, dans des conditions peu favorables, une erreur pouvant atteindre 15 mGal. Le développement en série suivant les fonctions sphériques jusqu'au 16 ème ordre et le calcul électronique des corrections dues aux courants à appliquer aux valeurs mesurées de la pesanteur, ont montré que le fait de ne pas tenir compte de ces corrections, peut apporter une erreur non négligeable, (0.1 dans le dénominateur de la formule de l'aplatissement terrestre). Des tables des coefficients de développement en série ont déjà été établies pour l'été et l'hiver.

J.G. MOURALEV a étudié les questions de l'application des relations intégrales entre les altitudes du quasigéoide, les latitudes et les longitudes géodésiques établies par Molodenskij.

Un procédé de calcul électronique pour les altitudes gravimétriques du quasigéoide et les déviations de la verticale d'après les formules intégrales de Stokes et de Vening Meinesz est décrit dans l'article de L.P. PELLINEN, V.A. TARANOV & A.J. CHABANOVA.

A.K. RAJINSKAS a évalué la précision de l'interpolation des déviations de la verticale astro-géodésiques, par les déviations de la verticale gravimétriques pour le territoire de Lithuanie et a établi des cartes des déviations de la verticale et du quasigéoide pour la partie occidentale.

J.D. ZONGOLOVIC a comparé les possibilités de la détermination des harmoniques du champ de pesanteur de la Terre à l'aide du levé gravimétrique terrestre et du mouvement des satellites artificiels de la Terre. Il donne des renseignements sur la détermination des harmoniques zonaux (avec les valeurs correspondantes de l'aplatissement de la Terre) et des harmoniques tesséraux (avec les valeurs correspondantes de la différence des demi-axes de l'ellipse équatoriale et de la longueur de son demi-grand axe).

S.V. GROMOV a obtenu la formule intégrale pour le potentiel perturbateur sur le géoide régularisé, par les anomalies du gradient vertical de pesanteur sur ce géoide.

A.A. KRGIJANOVSKAYA a employé les cartes qu'elle avait établies précédemment donnant les déviations gravimétriques de la verticale dans la région comprise entre les latitudes 53°15' - 54°15' et les longitudes 59° - 60°, pour calculer les différences des altitudes gravimétriques du quasigéoide par rapport à un des points de cette région. La précision des calculs est estimée égale à 20 cm.

G.J. MENAKER a calculé des abaques pour évaluer la correction de relief, avec une précision de 0,2 - 0,3 mGal, à partir des différences d'altitudes prises le long de profils radiaux autour de la station gravimétrique. Pour estimer l'effet des masses topographiques des régions les plus proches N.V. ZEMLAKOV propose d'employer la formule de l'attraction des couches minces horizontales sous forme de polyèdres comme la base pour les programmes de calcul électronique. Une méthode pour calculer les corrections topographiques avec un calculateur électronique est envisagée également par V.I. ARONOV et d'autres géodésiens.

R.B. BOURDUKOV a calculé les tables des valeurs normales de la pesanteur d'après la formule de Helmert (1901 - 1909) avec une précision de 0,01 mGal suivant l'argument des coordonnées planes rectangulaires de Gauss. Ces cartes s'étendent aux latitudes comprises entre 32° - 77°. En annexe sont données des tables de corrections pour transformer ces valeurs suivant la formule utilisée (formules de Cassinis, de Krassovsky ou de Zongolovic).

D.V. ZAGREBIN a rectifié une erreur de son ouvrage précédent: (Les travaux de l'Institut de l'Astronomie Théorique, 1952, n°1, p.87-222).

N.K. MIGAL a mis en doute la possibilité de déduire les déviations de la verticale à partir d'un levé gravimétrique local avec une précision plus grande que 0.5. Une réponse à cet article a été publiée par M.J. JURKINA & V.F. EREMEEV.

(Les références bibliographiques des auteurs cités ci-dessus sont incorporées dans la liste p. 28).

DETERMINATION de la FIGURE de la TERRE
par la
METHODE GRAVIMETRIQUE

Les articles traitant plus spécialement du calcul des déviations de la verticale et des données relatives à l'ellipsoide terrestre ont été classés sous des rubriques indépendantes.

A) RAPPORTS GENERAUX

64* - INT. ASSOC. GEOD. (Section V) - Report on theoretical and practical research on the problem of the figure of the Earth, made in the USSR, Hungary, Germany, Poland, Rumania and Czechozlovakia 1960-62.
Berkeley, I.A.G. Aug. 1963, 13 p.

General Report on theoretical and practical research work on the determination of the external gravity field and shape of the physical surface of the Earth carried out during the period 1960-62. List of the investigations published in 1960-62 in each country.

65 - BOMFORD G. - Geodesy. Oxford, London. 2ème edition 1962 : XVI + 561p. (1st edition ; 1952).
C.R. Geophys. 1963, v.XXVIII, n°6, p.1080-81. (J.L. SOSKE)

"The primary purpose of the second edition is to bring the book up to date. New and accepted methods in Geodesy developed since the Second World War and the necessity to point toward not yet firmly established new developments in Geodesy have increased the text from 451 to 561 pages between editions".

66 - CAPUTO M. - La pesanteur et la forme de la Terre
Bul. Géod. 1965, n°77, p.193-202.

Parmi les problèmes que l'ère spatiale a soulevés, il y en a deux qui sont liés au champ normal de la pesanteur de la Terre :

a) Le calcul du champ normal de la pesanteur et de la gravité dans l'espace autour de la Terre.

b) La compensation des paramètres du champ et de la forme de la Terre.

* Ces articles font suite à ceux indiqués dans le Bulletin d'Information (Avril 1962) p.149.

Il a semblé aussi intéressant d'étendre les formules de la théorie de Pizzetti-Somigliana, y compris la formule internationale, à l'espace autour de la Terre pour la pesanteur et la gravité, i.e., pour les champs contenant ou ne contenant pas la force centrifuge.

Des expressions approximatives pour cette compensation ont déjà été données par Helmert (1848), de Sitter (1938), Lambert (1960), Cook (1959) et d'autres. Les expressions finies ont été données par Pizzetti (1894), Somigliana (1929), Caputo (1963-64).

De quelques calculs préliminaires faits pour la compensation des paramètres adoptés par l'U.A.I., nous pouvons déduire que les corrections nécessaires sont très petites.

67 - EWING C.E. - Research and development in the field of geodetic Science. A.F.S. in Geophys. Aug. 1960, n°124, GRD, 58 p.

This Report briefly surveys significant research and development activities in geodesy, namely, arc measurements, electronic surveying, eclipses and occultations, moon camera, satellites and gravity measurements.

68 - GRUSINSKII N.P. - Théorie de la figure de la Terre. Manuel pour étudiants. Fizmatgiz. 1963. 446 p.

69 - HEISKANEN W.A. - How we can best benefit the physical Geodesy. Inst. Geod. Phot. & Cart. Columbus, 1962, 7 p.

Réponse au questionnaire du Dr. Tengström concernant la meilleure méthode dans le présent et le futur pour déterminer la figure de la Terre. Résumé sur ce sujet des travaux de L'Inst. Geod. Phot. & Cart."of Ohio State University ".

70 - HEISKANEN W.A. - Le basi geodetiche campione e le dimensioni della Terra. Boll. di Geod. e Sci. Affini, Ist. Geogr. Mil. 1963. Anno XXII, n°4, p.451-474.

71 - HEISKANEN W.A, K. ARNOLD, A. BJERHAMMAR, K. LEDERSTEGER, H. MORITZ. R. RAPP, E. TENGSTROM, U. UOTILA. - Physical Geodesy. Author's individual Reports. Berkeley. 13th G.A. of I.U.G.G. Aug. 1963.

72 - HEISKANEN W.A. K. LEDERSTEGER. M. BURSA. - General Reports. Berkeley. 13th G.A. of I.U.G.G. Aug. 1963.

73 - HEISKANEN W.A. & H. MORITZ - Methods of Physical Geodesy.
Inst. Geod. Phot. & Cart. Columbus. 1964, Rep. n°32, 30 p.

"A brief review of some geodetic methods on the determination of the gravitational field of the Earth. After a historical introduction, it deals with the gravimetric and astro-geodetic methods and with their combined applications. It discusses those parameters of the Earth which are needed in these applications, e.g., the equatorial radius, the equatorial gravity, the harmonic coefficients, the flattening etc...

A brief Section is devoted to the effect of these parameters on the gravitational perturbing forces at satellite altitudes. A rather extensive bibliography completes his Report."

74 - HEISKANEN W.A. H. MORITZ. & I.I. MUELLER - Research directed toward a feasibility study dealing with the integration of gravity data.
Inst. Geod. Phot. & Cart. Columbus. 1964, Rep. n°31. 26 p.

"This summary deals with the following individual papers and is updated, since they were written a year ago :

1) Methods of physical geodesy
2) A review on close satellite geodesy
3) The geodetic applications of satellites
4) Satellite perturbations due to zonal gravitational harmonics
5) On the accuracy of spherical harmonics and orbital prediction."

75 - HEISKANEN W.A. - Present problems of Physical Geodesy
Pub. Isost. Inst. Helsinki. 1965, n°49, 35 p.

"Physical geodesy can solve the most important problems of the present geodesy alone or with the astro-geodetic and satellite geodesy - if the gravity anomalies around the Earth's surface are available. In fact, most part of the Earth is gravimetrically unsurveyed. Therefore we must fill the gravity anomaly gaps either by new gravity anomalies to the unsurveyed areas by statistical methods, or geophysically from the topographic and isostatic compensating masses with or without gravity observations.

The author has after the historical review of the Physical Geodesy described the last years achievement of the Columbus group on this most important part of Physical Geodesy. He has also emphasized the global significance of the isostasy and of isostatic anomalies in Physical Geodesy. The isostatic compensation makes the undulations of the geoid and of other geopotential surfaces small, the isostatic anomalies are most representative in the extrapolation of the gravity anomaly field to unsurveyed areas. Finally the geoids of Hirvonen,Tanni, Zongolovic, Heiskanen, Uotila and Kaula are presented".

*
76 - WILLIAMS O.W. - A Compendium of papers in the fields of Geodesy and Planetary Geometry prepared at AFCRL during 1962.
AFCRL.63-876, Aug. 1963, 232 p.

...

Chap. V : Three dimensional geodesy and an Earth centered coordinate system.

...

Chap. X : Geodesy - promise and progress.

"This Compendium presents a series of technical papers prepared by scientists of Air Force Cambridge Research Laboratories during 1962, in the fields of geodesy and planetary geometry. These papers describe research activities in the areas of satellite geodesy (in particular, Project ANNA), rocket-flare triangulation, airborne gravimetry, absolute and relative gravity, laser geodesy, and solenodesy. Illustrations show recent geodetic instrumentation developments and technique configuration.

77 - WILLIAMS O.W. - A Compendium of papers in the fields of Geodesy and Planetary Geometry prepared at AFCRL during 1963.
AFCRL.65-14, Jan. 1965, Sp. Rep. n°18, p.29-39.

...

Chap. III : Geodetic research in the space age.

"Though the space age has placed very stringent demands upon the geodesist for continual improvement in the certainty with which we know the size and shape of the earth, this same space age has brought new techniques and technologies which appear to have considerable potential in the field of geodetic measuring. Use of the artificial satellite as a geodetic data acquiring tool is an actuality ; measuring gravity from an aircraft is a reality ; the laser offers more directable, measurable light sources for long-range geodesy. The benefit to geodesy from application of new techniques can be considerable, the need for these techniques is already considerable".

78 - ZAKATOV P.S. - Cours supérieur de Géodésie.
Uceb. Geod. Vuzov. Fakult. 1964. (texte russe)

*79 - MOLODENSKIJ M.S, V.F. EREMEEV & M.I. JURKINA - Method for study of the external gravitational field and figure of the Earth.
Trudy Cniigaik, 1960, n°31, Moscou.-Traduction anglaise, 1962, Off. Tech. Serv. US Dept. of Comm. Washington.
C.R. in Geophys, 1963, v.XXVIII, n°4, p.672-673. (R.A. Hirvonen).
Deutsche Hydr. Z, 1965, 18, h.3, p.134-135. (V. Fleischer)

B) ETUDES THEORIQUES

80 - ARNOLD K. - Eine deifache Ableitung des gravimetrischen Zusatzgliedes. Gerlands Beitr. Geophys. 1965, 74, H.3, S.207-211.

There is given a short derivation of the gravimetric correction term of physical geodesy.

81 - ARNOLD K. - The boundary-value problem of physical geodesy and its solution. Studia Geophys. & Geod. 1965, v.9, n°2, p.113-118.

82 - BHATTACHARJI J.C. - Modified form of Earth model gravity anomaly for use in Stokes's integral.
Geophys. J. Dec.1965, v.10, n°3, p.221-229.

Graaff Hunter's corrected formula to reduce the observed values of gravity to the surface of the Earth model, has been further simplified and the surface anomaly has been modified accordingly for introduction into Stokes's integral without recourse to any isostatic assumption.

83 - BJERHAMMAR A. - The Geosphere as the fundamental surface in geophysical geodesy.
Note presented to Berkeley (I.U.G.G.) Aug. 1963.

The author proposes the reduction of gravity to a sphere in order to solve the boundary value problem of geodesy in a way that does not depend on assumptions about the outer masses.

84 - BJERHAMMER A. - On the determination of ths shape of the geoid and the shape of the Earth from an ellipsoidal surface of reference.
Text presented to Berkeley (I.U.G.G.) Aug. 1963, 35 p.

The solution of this problem, the improvment of Stokes's formula, by Sagrebin contains an error. This error was corrected and a new solution was found.

85 - BJERHAMMAR A. - A general World geodetic system without reference surface. Text presented to Berkeley (I.U.G.G.) Aug. 1963, 6 p.

"The main feature of this system is that a reference figure is not used as a part of the solution, either in the geophysical or in the geometrical aspects. To obtain the solution, the gravity anomalies are replaced by the directly determined gravity values. Instead of having geodetic anomalies, we get a geodesy which only deals with directly observed quantities. Furthermore all geometrical observations are used without reduction to any reference figure...".

86 - BJERHAMMAR A. - <u>A new theory of gravimetric geodesy.</u>
Royal Inst. of Technology. 1963. Stockholm.
C.R. in Studia Geophys. & Geod. 1965, v.9, n°2, p.112.

87 - BIRO P. - <u>Eine modifizierte Lösung der Randwertaufgabe der geodätischen Gravimetrie auf der Erdeberfläche.</u>
Studia Geophys. & Geod. 1965, v.9, n°2, p.119-123.

88 - BRAGARD L. - <u>Method to determine the shape of the topographical Earth surface by means of gravity measurements on that surface solving two integral equations.</u>
Studia Geophys. & Geod. 1965, v.9, n°2, p.108-111.

En appliquant le théorème de Green et un de ses cas particuliers, on obtient une équation de Fredholm donnant la distance de la surface de niveau extérieure à la terre comptée normalement à partir de la surface topographique en fonction de mesures de pesanteur sur ces deux surfaces.

En considérant une surface référentielle de niveau contenant un volume égal à celui de la Terre et de même masse, on obtient une seconde équation de Fredholm donnant la distance de la surface extérieure à la référentielle comptée normalement à la surface topographique.

En résolvant les deux équations intégrales, il est possible d'obtenir la distance de la surface topographique à la référentielle comptée normalement à la première.

89 - BRAGARD L. - <u>Methods to determine the shape of the topographical Earth surface by successive regional determinations of this shape in function of regional gravity measurements or in function of these and regional vertical gravity gradient measurements.</u>
Typewritten text, presented to the I.G.C, Paris, Sept.1965, 12 p.
Medd. Från Geod. Inst. Uppsala, 1966, n°9.

Applying various particular cases of Green's formula twice two Fredholm's equations are obtained, which admit each a unique solution. They allow to get the distance between a vault of the topographical Earth surface and the corresponding reference vault on one side, in function of gravity values and on the other side in function of these and vertical gravity gradient values on these vaults. Using the same process to successive vaults of the topographical Earth surface so as to cover it entirely, it is possible to determine its shape knowing only on one side, the regional gravity values and on the other side these and the regional values of vertical gravity gradient, i.e, the values on successive vaults, without being obliged to conserve the same external level surface from one to another region.

90 - BROVAR V.V. - Solution des quelques problèmes extérieurs limites pour la surface terrestre.
Izvest. Vys. Zav. Geod. Aerofot. 1963, n°1, p.55-72.

91 - BROVAR V.V. - Calcul du potentiel perturbateur et de ses dérivés au-dessus de la Terre.
Izvest. Vys. Zav. Geod. Aerofot. 1963, n°3, p.57-63. (texte russe)

92 - BROVAR V.V. - Sur les solutions du problème limite de Molodenskij.
Izvest. Vys. Zav. Geod. Aerofot. 1963, n°4, p.129-137.(texte russe)

93 - BROVAR V.V, V.F. EREMEEV, N.P. MAKAROV, L.P. PELLINEN, B.P. SIMBIREV, M.J. JURKINA. - A propos de la détermination du champ gravifique extérieur et de la figure de la Terre.
Geod. Kart. 1963, n°10, p.74-76. (texte russe)

94 - BROVAR V.V. - Potentiels extérieurs généralisés.
Izvest. Vys. Zav. Geod. Aerofot. 1964, n°1, 18 p. (texte russe)

95 - BROVAR V.V. - Fonctions harmoniques fondamentales...
Izvest. Vys. Zav. Geod. Aerofot. 1964, n°3, p.51-61.

96 - BROVAR V.V. - On the solutions of Molodenskij's boundary value problem.
Bul. Geod. 1964, n°72, p.167-174.

"The determination of the disturbing potential from mixed anomalies of gravity on the Earth's surface can be made to amount to the solution of different integral equations for auxiliary densities. In seeking new ways of representing the disturbing potential and new integral equations corresponding to the former one can have recourse to the solution of simpler boundary problems (e.g, Dirichlet's or a modification of Neumann's). Two new ways of solving Dirichlet's problem have been found, and on their bases two new solutions of Molodenskij's problem are given. On a sphere both integral equations become elementary algebraic ones. Therefore their solution is simpler than that of Molodenskij's basic equation".

97 - BURSA M. - On the practical computation of the quasigeoidal characteristics applying Molodensky's formulae in the first approximation.
Stud. Geophys. & Geod. 1964, v.8, p.109-119.(russian text, english abstract)

"Molodenskij's last solution of the fundamental integral equation of physical geodesy, based on a development introducing a small parameter (a function of the density of a single layer) is convenient for practical computation. If the terms of the zero approximation and of the first approximation are taken into account and the terms of the second approximation are neglected, the quasigeoidal height and the components of the deflection of the vertical at the physical surface of the Earth can be expressed (see original text). The corrective terms contain the correction G_I for gravity anomalies depending on the topography. The problem of the practical computation of G_I is discussed as well as possibilities for constructing maps with the lines G_I = const. The method where only the integrals $\int (g - \gamma)\, r^{-2} dr$, $\int H\, r^{-2} dr$ are interpolated, is recommended as the most convenient. The individual values H_o and $(g - \gamma)_o$ must be determined at the characteristic points of the physical surface of the Earth as well as of the gravity field. Two formulae are recommended for the practical computation of G_I depending on whether the values of terrain correction have been calculated before or not."

98 - BURSA M. - On practical application of the solution of Molodenskij's integral equation in the first approximation.
Studia Geophys. & Geod. 1965, v.9, n°2, p.144-149.

"In the first table, the author gives "the numerical results from 4 models formed on the basis of topographic massives of the High Tatras and Saxon Erzgebirge. The model E is not connected with the actual topography, and represents a cone with $H_q - H_o = r$. It has been formed only as a matter of fact, that in this case methods I and II (weight function $p(r) = r^{-2}$) grant the exact result by arbitrary n, but methods III and IV (weight function $p(r) = 1$) are considerably inaccurate".

... In conclusion "the method I (with $B_k > 0$) seems to be the most suitable. The respective values of abscissae r_k and template coefficients B_k for two kinds of terrain (mountainous and flat areas) suitable for practical use, are given in the second table".

99 - BURSA M. - Molodenskij's formulae and their application to the Test-area work. Typewritten text. July 1965, 3 p.
Medd. Från Geod. Inst. Uppsala, 1966, n°9.

After having reminded the known Molodenskij's relations, the author insists on the necessity of detailed gravity information around the points where the surface data are to be calculated (at least 1 gravity point per 3 - 5 km^2).

100 - CALJUK T.N. - Sur la détermination préalable des réductions constantes dans la théorie de H.K. MIGAL.
Nauk. Zapiski. Louvosk. Polit. Inst. 1962, n°85, p.83-85.

101 - DECKER L.B. - Gravity disturbance components from a spherical harmonic expansion of the disturbing potential.
ACIC Tech. Paper n°18, Saint-Louis, April 1965, 18 p.

The solution of the basic "differential equation of gravimetry" is the disturbing potential, T. Gravity disturbance components in a local orthogonal coordinate system are obtained from the gradient of T where T is expressed in spherical coordinates r, ϕ, λ. Gravity disturbance components in rectangular Earth-centered, Earth-fixed (ECEF) and Earth-centered, inertial (ECI), reference systems are provided by a coordinate transformation.

102 - EREMEEV V.F. - A method for computing the disturbing potential plumb-line deflections and Stokes's constants on the basis of Molodenskij's integral equation for the disturbing potential.
Studia Geophys. & Geod. 1965, v.9, n°2, p.128-136. (russian text, english abstract)

" Formulae were obtained for expressing corrections of Stokes's approximations of the disturbing potential, plumb-line deflections, and coefficients of expansion of the disturbing potential in terms of spherical harmonics. When used for the computation of plumb-line deflections in any terrain conditions, these formulae will ensure relative accuracy of order of the Earth's flatteining if the distance between the given and a current point is greater than 300 km. In conditions of an undisturbed relief, this distance may be diminished and reduced to zero in flat areas. In difficult terrain conditions inside the central zone, the application of Molodenskij's equation for the disturbing potential together with the equation for the single layer density is necessary. The projections of the elementary areas onto the reference sphere may be formed by arcs of meridians, parallels and diagonals of the corresponding trapezia. Besides the gravity anomalies and Stokes's approximation values of the disturbing potential, the mean slope of the Earth's surface, and the mean square value of this slope must be known for the elementary area in question".

103 - EREMEEV V.F. & M.J. JURKINA. - Methods of calculation based on Molodenskij's integral equation for the disturbing potential.
Trudy Cniigaik. 1965, n°157, p.3-45. (russian text, english abstract)

"Making use of the method applied by Molodenskij in deduction of his integral equation for the disturbing potential T, an expression was obtained for the T - derivative along a tangent τ to the physical surface of the Earth. In this expression, r - is the distance between the examined and the current points on the physical surface of the Earth, g - the measured value of gravity, y - that normal, v - a normal to the surface of reference, α - the angle of tilt of the surface S with respect to that of reference, $\bar{T}$ - the value of T at the point under examination. This expression permits to represent the plumb-line deflection on the physical surface of the Earth through gravity anomalies and the disturbing potential.

Ways for computing the disturbing potential and plumb-line deflections at points on the physical surface of the Earth as well as those of calculating the Stokes's constants (coefficients of the disturbing potential expansion in terms of spherical functions), based on the above-mentioned Molodenskij's integral equation, are described. From the enumerated values the Stokes's approximations were isolated. For the purpose of calculating the corrections to these approximations, the physical surface of the Earth should be represented by sides of a polythedron ; the projections of the sides on the sphere of reference are spherical triangles. The heights of the sides are determined by the formula :

$$h = a \vartheta + b \lambda + c,$$

where ϑ and λ are the polar distance and the longitude, respectively, and the coefficients a, b and c, are to be determined from the heights h at the angles of the sides.

If the distance between the point under investigation and the current point is over 300 km, or else the terrain around the point under investigation is sufficiently flat, then the disturbing potential and plumb-line deflections can be expressed by different formulae. The first Stokes's constants are represented by formula (71, see text).

An investigation carried out on an Earth's model in the form of a cone situated on the plane of reference has shown that the Stokes approximation to the disturbing potential is not sufficient to improve, with the help of the given expression, the Stokes approximation to the plumb-line deflections, i.e, it can not serve to rise the accuracy of calculations from the Vening-Meinesz formula of the influence of the area near the point under examination in a mountainous region".

104 - GRAAFF-HUNTER de J. - The external gravity field of the Earth in terms of gravity anomalies at ground level.
1963, 2 typewritten pages. A fuller paper is under preparation.

105 - GRAAFF-HUNTER de J. - Imperfections in finding Earth shape from gravity and some suggestions for improvement.
Bull. Géod. 1958, n°47-48, 28-47.
Bull. Géod. 1960, n°56, 191-200.
C.R. in Studia Geophys. & Geod. 1965, v.9, n°2, p.165-169.

106 - GROMOV S.V. - Détermination de la figure de la Terre par les anomalies du gradient vertical de la pesanteur.
Ucen. Zap. Leningradsk. Gosudar. Univ. 1962, n°307, p.234-242.

107 - GROTEN E. - On the accuracy of the three components $\partial \Delta g / \partial x_i$ at low elevations derived from gravity anomalies.
I.G.P.C. Columbus. AFCRL 65-521. Rep. n°48, 16 p.

The error in the three components of the gravity anomaly gradient as derived from surface gravity anomalies is studied. Error propagation, due to the errors in Δg at the surface is considered where the Earth's surface is replaced by a plane. For low elevation h the same error for all three components is found. For $h = 0$, only the vertical gradient is considered.

108 - GROTEN E. - Accuracy of gravity gradients at elevations.
I.G.P.C. Columbus. AFCRL 65-523. Rep. n°50, 14 p.

Formulas are derived for the computation of accuracy of the three components of gravity disturbance gradient at elevations as determined from surface anomalies. The Earth is replaced by its tangent plane.

109 - GRUSINSKII N.P. & M.U. SAGITOV - Influence des courants marins sur l'étude du champ de pesanteur extérieur.
Sbor. "Mor. Grav. Issl". Moskov Univ. 1963, Bul. n°2, p.105-114.

110 - HIRVONEN R.A. - Statistical analysis of gravity anomalies.
I.G.P.C. Columbus. 1962, Rep. n°19, 26 p.
Pub. Isost. Inst. Helsinki, n°37.
(Ann. Acad. Sci. Fen. Ser. A, III, Geol. Geogr, n°58)

In the first part of this paper the author continues his research on the statistics of gravity anomalies. New and independant methods for computing the statistical functions E_S and G_S are described. E_S, the error of representation, can be obtained using a map with iso-anomaly curves. G_S, the root mean square anomaly, may be computed if the covariance function, G_d, is given empirically. The formulae are illustrated by some numerical examples.

The second part treats the continuation of the gravity anomaly field to different elevations. An earlier integral formula, expressing them in terms of gravity anomalies at ground level, can be simplified and adapted for electronic computing. For manual computation the statistical method described above is used to find a template which gives satisfactory accuracy with least amount of labor. The derived methods for the manual and high speed computation for the extension of the gravity field in unsurveyed areas and its continuation to high elevations have great theoretical and practical significance. Comparison of different methods continues.

111 -HIRVONEN R.A. & H. MORITZ - Practical computation of gravity at high altitudes. I.G.P.C. Columbus. May 1963, n°27, 88 p. (Two parts written respectively by each author)

Methods for computing the gravity vector outside the Earth are presented and compared. For the purpose of computation, gravity is split up into normal gravity and gravity disturbance. The report first gives a practical method for computing normal gravity. For gravity disturbances, different methods are described and compared by means of accuracy studies (on influence of distant zones, etc...), finally a practical computation procedure is given.

112 - IZOTOV A.A. - On the détermination of the shape and dimensions of the Earth from observations of artificial satellites. Studia Geophys. & Geod. 1965, v.9, n°2, p.201-206.

The height of the quasigeoid over the surface of a reference ellipsoid at an initial point can be calculated from the shape rectangular co-ordinates of that point and from the height above sea-level, which were derived from the observations made by artificial satellites.

Using these data we can form equation enabling the determination of the parameters of a general Earth's ellipsoid and elements of position and orientation or a reference ellipsoid in connection with the gravity point of mass and the inertia Earth's axes. The derived equations of the measurement of the grade of general type are correct even when using the results of astronomical or astronomical-gravimetrical levelling.

The task of constructing a world system of coordinates developed from both artificial satellite observations and from classical methods of geodesy is necessarily connected with the task of determining the parameter of the general Earth's ellipsoid.

113 - JURKINA M.J, N.P. MAKAROV & V.F. EREMEEV. - On the present state of the theory of study of the physical surface of the Earth.
Trudy Cniigaik. 1960, n°139, p.45-59.

114 - JURKINA M.J. - Computation of the first and the second vertical derivatives of gravity
Studia Geophys. & Geod. 1965, v.9, n°2, p.101-107.

Following Molodenskij's suggestions anomalies of the vertical gradient of gravity were used to achieve a greater accuracy in the determination of the figure of the Earth by gravimetrical methods. The existing methods of computing this quantity do not take into account inclinations of the physical surface of the Earth. Using the Laplace equation, the second derivative $\partial^2 T/\partial \nu^2$ of the disturbing potential T is expressed by the second derivatives of T along the tangents τ_1 and τ_2 to the physical surface of the Earth in mutually perpendicular planes and by the derivatives of gravity anomalies. The derivatives $\partial^2 T/\partial \tau_1^2$ and $\partial^2 T/\partial \tau_2^2$ have been determined using the Molodenskij method of solving his integral equation for the single layer density. In the zero approximation, the Noumerov formula was obtained ; however, the results obtained using this formula should be referred to the physical surface of the Earth, not to the Listing Geoid. The correction of the first approximation is given by formula (16, see text). The second vertical derivative of gravity anomalies can be determined using the expression (20, see text).

115 - JURKINA M.J. - Calculation of the first and the second vertical derivatives of gravity from maps of gravity anomalies.
Trudy Cniigaik. 1965, n°157, p.116-124. (russian text, english abstract)

On the basis of the solution proposed by Molodenskij of his integral equation for density of a single layer on the physical surface of the Earth (the said layer being substituted for the attraction of all the anomalous masses in the interior of the Earth), expressions were obtained for determining the second vertical derivatives of the disturbing potential.

116 - JURKINA M.J. & G.A. ALEKSASHINA - Estimating on a model of the Earth some correcting members to Stokes's approximation for coefficients of expansion of the disturbing potential in terms of spherical functions.
Trudy Cniigaik. 1965, n°157, p.58-65. (russian text, english abstract)

Corrections to the Stokes approximation to terms of the 2nd, 8th and 16th order of expanding the disturbing potential were estimated using as an example the case of determining the influence of a mountain range girding a spherical Earth. Heights of the vertical sections of this range were assumed to be equal to those taken from hypsometrical maps of Tibet, the Hymalaya, and Caucasus. The surface of the range was approximately equal to the area of all the Earth's mountain regions, it is also shown that the accuracy of computing the Stokes's approximation may diminish noticeably if the surface of elementary areas of numerical integration is enlarged. The results of computation, expressed in meters, are represented in a table.

117 - JURKINA M.J. & A.B. STAROSTINA - Using a model of the Earth to check the principle of numerical integration as applied to calculations on the basis of Molodenskij's integral equation for the disturbing potential.
Trudy Cniigaik. 1965, n°157, p.47-57. (russian text, english abstract)

The validity of the principle of numerical integration, described in an article by Eremeev and Jurkina, was checked on an Earth model in the form of a cone situated on the plane of reference. The value of the disturbing potential in the vertex of the cone was calculated. The surface of the cone was approximately represented by plane sides - less accurately in the first version, and more accurately - in the second (See the concluding table where the results for some sections of the mode are reported, and the last line gives the total influence).

118 - KRIZANOVSKAJA A.A. - Sur une caractéristique numérique locale de la figure de la Terre.
Zap. Leningr. Gor. Inst. 1964, 43, n°3, p.100-113. (texte russe)

119 - LEDERSTEGER K. - Der Physikalische Zusammenhang zwischen der statischen Abplattung und dem inneren aufbau der Erde auf der Hypothetischen Grundlage einer sechsparametrigen Geichgewichtsfigur.
Geof. Pura e Appl. Milan. 1962, Bd. 51, S. 1-27.

A general theory of multiparameter figures of equilibrium is developed. The condition of equilibrium is formulated as folows :

1) The density-law is strictly individual.

2) Each figure of equilibrium consists of several parts separated by surfaces of discontinuity, each of which has the density-law of one parametric figures of equilibrium or is homogeneous.

3) Peeling off the figure to the lowest surface of discontinuity results a zero or a one parametric core. For the normal spheroid of the Earth a three-partite model consisting of a homogeneous water-cover, a heterogeneous mantle and of such a core is taken as basis. After peeling off the ocean a four-parametric solid Earth remains, assuming the depth of the surface of the core in 2900 km. As for an unequivocal solution two data are lacking, we find solutions the domain of which is separated by three linear series of boundary-figures : a series of figures with homogeneous core and heterogeneous mantle, a series of figures with homogeneous mantle and heterogeneous core and a series with vanishing difference of density on the surface of the core.

The three corner points of this field are represented by following figures :

a) A Wiechert model consisting of a homogeneous mantle and a homogeneous core ; this figure has the maximum of moment of inertia C.

b) The boundary figure B with homogeneous core and vanishing difference of densities, which has the minimum of C.

c) The boundary figure with homogeneous mantle and also vanishing difference of densities on the surface of the core. Within the empirical uncertainty the moment of inertia C calculated from the dynamic and static flattening of the real Earth coincides with the minimum value of figure B. Therefore, it seems that the real Earth is distincted by the minimum moment of rotation, in which case simultaneously the core must be homogeneous and the difference of density on the core's surface must vanish".

120 - LEDERSTEGER K. - Les satellites artificiels et les fonctions de masse de la Terre.
Acta Technica, Hongrie. t.43, Fasc. 1-2, p.231-258.

..."L'auteur expose la relation entre les anomalies de l'orbite des satellites artificiels et les fonctions de masse figurant dans la série du potentiel terrestre.

... La contradiction apparente entre l'aplatissement statique et dynamique s'explique tout simplement par le fait que la figure normale de la Terre n'est pas une forme à un seul paramètre. Par là, on peut aussi réfuter ce résultat erroné déduit de la théorie classique, qui veut qu'il existe un écart considérable entre l'aplatissement effectif et hydrostatique."

121 - LEDERSTEGER K. - Die Neubegründung der Theorie der sphäroidischen Gleichgewichtsfiguren und das Normalsphäroid der Erde.
Osterr. Z. Vermes. 1964. (Hundertjahrfeier Osterr. Kommiss. Int. Erdm. 23 bis 25 Oktober 1963). S. 31-125.

122 - LEDERSTEGER K. - Das allgemeine Niveausphäroid in Näherung achter Ordnung. Osterr. Z. Vermesswes. 53 Jg. 1965, H.5. 8 S.

123 - LEDERSTEGER K. - Die Gleichgewichtsfigur und das Normalsphäroid der Erde. Z. Vermesswes. 90 Jg. 1965. H.11, S.385-393.

124 - LEDERSTEGER K. - Critical notes on the mass-functions derived from orbital perturbations of artificial satellites.
Typewritten text, presented to the Symposium of Athens. 1965, 7 p.

125 - LEDERSTEGER K. - The equilibrium figure and the normal spheroid of the Earth.
Typewritten text, presented to the Symposium of Athens. 1965, 14 p.

126 - LEVALLOIS J.J. - La réhabilitation de la géodésie classique et la géodésie tridimensionnelle.
Bul. Géod. Juin 1963, n°68, p. 193-199.
Le ler texte provisoire a déjà été indexé. (Bul. Inf., Avril 1962, p.56)

127 - MARYC M.I. - Détermination de la figure de la Terre avec un calcul des quantités infinitésimales du 3ème ordre.
Izvest. Vys. Zav. Geod. Aerofot. 1963, n°3, p.65-74.

128 - MARYC M.I. - On determination of the figure of the Earth without applying of normal field.
Bul. Géod. 1964, n°73, p.261-264.

"We shall consider briefly the fundamental principles of the theory of the determination of external levelled surface of the planet, taking into consideration the method of the determination of the figure of the Earth, proposed by N.K. Migal (1949) in which the concept of normal gravitation field has been discarded...".

129 - MAZZON C. - Le espressioni generali dello scostamento del geoide e della condizione di Villarceau.
Politecnico di Milano. 1965, n°164, 15 p.

On a déduit la formule rigoureuse de l'écartement d'une superficie générale d'une autre de référence, sur laquelle on avait définit la congruence des normales à la première sans introduire aucune limitation pour la déviation et on a généralisé la condition de Villarceau que doivent satisfaire les composants de la déviation relative à la superficie de référence choisie.

130 - MIGAL N.K. - Sur la précision de la détermination des constantes de réduction, des hauteurs du géoide et des déviations de la verticale.
Nauk. Zap. Louvosk. Polit. Inst. 1962, n°85, p.54-67.

131 - MIGAL N.K. - A propos de l'article de I.F. MONIN "Sur une méthode pour l'étude de la figure de la Terre, sans utilisation du champ normal".
Izvest. Vys. Zav. Geod. Aerofot. 1963, n°6, p.141-144.

132 - MOLODENSKIJ M.S. - Uber die Aufgabe der Gradmessungen .
Bull. Géod. 1962, n°64, p.137-144.

133 - MOLODENSKIJ M.S. - Lösung der Integralgleichung zur ermittlung der Erdfigur.
Bull. Géod. 1962, n°64, p.181-187.

134 - MOLODENSKIJ M.S, V.F. EREMEEV, M.J. JURKINA. - An evaluation of accuracy of Stokes's series and some attempts to improve his theory.
Trudy Cniigaik. 1962, n°145, p.2-21. (russian text, english abstract)

"Five different variants of expanding the disturbing potential in terms of spherical functions were carried out for an Earth model in the shape of a sphere girded along the equator with a tor, the said tor being covered on both sides with "deckings" (the greatest inclination of the model surface is near 10°)

...

5) By introducing corrections $\Delta_2 A$ to the coefficients $A + \Delta_1 A$ of the fourth variant to account for corrections G_2 of second approximation of numerical solution of the same equation."

...

This text is similar to the text published in Bul. Géod. 1962, n°63, (see Bul. Inf. April 1962, p.56-57). The 5th variant was introduced further in this text.

135 - MONIN I.F. - Sur les problèmes fondamentaux concernant la géodésie et la gravimétrie.
Nauk. Zap. Louvosk. Polit. Inst. Ser. Geod. 1961, n°6, p.43-93.

136 - MONIN I.F. - Détermination du géoide régularisé avec un calcul des quantités du 3ème ordre.
Nauk. Zap. Louvosk. Polit.Inst. 1962, n°85, p.86-93.

137 - MONIN I.F. - Sur la détermination de la figure de la Terre avec une erreur relative de l'ordre du carré de l'aplatissement de l'ellipsoide terrestre.
Izvest. Vys. Zav. Geod. Aerofot. 1962, n°4, p.101-115.

138 - MONIN I.F. - Sur la théorie de la régularisation du géoide.
Izvest. Vys. Zav. Geod. Aerofot. 1963, n°6, p.91-94.

139 - MONIN I.F. - Au sujet de la détermination de la figure topographiq de la surface terrestre avec une erreur relative.
Astr. ZH. 1963, 40, n°3, p.571-578.

140 - MONIN I.F. - Sur les calculs des anomalies de pesanteur, des haute quasigéoidales et des déviations de la verticale.
Dokl. Akad. Nauk. USSR. 1963, n°5, p.596-599.

141 - MONIN I.F. - Au sujet de la théorie de la figure de la Terre.
Dokl. Akad. Nauk. USSR. 1963, n°7, p.879-882.

142 - MONIN I.F. - A propos du problème de la détermination de la forme topographique de la surface terrestre.
Dokl. Akad. Nauk. USSR. 1963, n°9, p.1177-1181.

143 - MONIN I.F. - Solution de l'équation intégrale de M.C. Molodenskij, sur la détermination de la figure physique de la surface terrestre, avec un calcul du 3ème ordre.
Astr. ZH. 1965, 42, n°1, p.183-189.

144 - MORITZ H. - Some remarks on the questions of Dr. TENGSTROM for the Paris Conference 1962.
Ohio State Univ. 1962, 2 typewritten pages.

> From a logical point of view no method seems to be definitely preferable... With our present gravity material, therefore, isostatic anomalies seem to yield most accurate results. Or, if we use free air anomalies, we should in any case use Bouguer or isostatic reduction as an interpolation or (zero isostatic anomaly) prediction method. Free air anomalies have of course, to be used for computation of the external gravity field of the Earth.

145 - MORITZ H. - Studies on the accuracy of the computation of gravity in high elevations.
I.G.P.C. Columbus. 1962, Rep. n°21, 47 p.
Publ. Isos. Inst. Helsinki, 1962, n°38.
or Ann. Acad. Sci. Fenn., Ser. A, III, Geol. Geogr., n°59, 37 p.

The study of gravity in high elevations has gained practical importance during the last years. It is necessary, esoecially in connection with checking airborne gravimetric surveys, not only to compute these gravity values from gravity measured on the Earth's surface, but also to know the accuracy obtainable.

In the following a general theory of accuracy of these computations is sketched and numerical values given. Further, investigations are to follow...

After the theoretical foundations are summarized, some detailed investigations concerning different computational methods are given. The errors arising from representation, interpolation and from replacing the integrals by sums are studied, and finally a possibility of computing the influence of topographical irregularities and of the neglected outer zones is outlined.

The results may provide an idea of the accuracy obtainable by different methods.

146 - MORITZ H. - Interpolation and prediction of gravity and their accuracy. I.G.P.C. Columbus. 1962, Rep. n°24, 70 p.

In Section A, several methods for interpolation and extrapolation of gravity are described, such as geometrical interpolation, zero anomaly, representation, and least squares prediction.

General formulae for the accuracy of any prediction method are deduced in Section B. Least squares prediction, which is the most accurate one, permits a purely numerical processing of gravity anomalies and seems to be excellently suited for electronic computers. The accuracy of the usual prediction methods is shown generally and by means of numerical examples and the influence of the configuration of gravity stations is studied.

Section C deals with the prediction of mean anomalies. The important correlation of free air anomalies with elevation is the subject of Section D. Least squares prediction is extended to this case, giving matrix formulae which can be used for machine computations. The validity of the usual method of Bouguer anomalies is investigated, including the error committed by using a wrong Bouguer gradient.

An appendix deals with the error covariance function.

147 - MORITZ H. - Interpolation and prediction of point gravity anomalies
Publ. Isost. Inst. Helsinki. 1963, n°40, 32 p.
(Ann. Sci. Fenn. Ser. A., III, n°69)

Cet article est semblable à l'article précédent (n°146) mais comporte quelques paragraphes en moins.

148 - MORITZ H. - 1) A statistical method for upward and downward continuation of gravity.

2) On the arrangement of gravity station for the upward continuation.
I.G.P.C. Columbus, 1962, Rep. n°25, 27 p.

1) By replacing the actual gravity anomaly function by a suitable statistical model, it is possible to obtain a function of elevation H so that multiplying any free air gravity anomaly at ground by this function yields an approximate value for the corresponding gravity anomaly at height H. This method which has been tested numerically can be applied for upward and downward continuation of gravity. Another method for downward continuation is sketched.

2) For moderate elevations, especially for airborne gravimetry, the influence of the arrangements of gravity station and of the neglected outer zones is estimated. Based on this, a specific arrangement is proposed.

149 - MORITZ H. - On the theory of errors of physical geodesy, (gravitational field of the Earth and satellite orbits).
I.G.P.C. Columbus, 1963, n°26, 20 p.

General formulae for error propagation in linear integral transformations are given and applied to the accuracy of the computation of the gravity vector at high altitudes from surface gravity anomalies and to the accuracy of satellite orbits computed therefrom.

150 - MORITZ H. - On the accuracy of spherical harmonics and orbital predictions. I.G.P.C. Columbus, 1964, Rep. n°36, 18 p.

"This report is a combination of 2 shorter reports "On the accuracy of the computation of spherical harmonics from gravity anomalies" and "On the use of gravity measurements for orbital prediction and its accuracy".

The needed formulae for the gravity potential are derived and the standard error of the obtained coefficients of spherical harmonics are handled. Satellite orbits are influenced by the irregularities of the Earth's gravity field, which are usually expressed in terms of a development in spherical harmonics.

151 - MORITZ H. - An extension of error theory with application to spherical harmonics.Publ. Isos. Inst. Helsinki, 1964, n°44, 11 p.
(Ann. Acad. Sci. Fenn., Ser. A, III, n°75)

152 - MORITZ H. - The boundary value problem of physical geodesy.
I.G.P.C. Columbus, 1964, n°46, 66 p.
Publ. Isos. Inst. Helsinki, 1965, n°50.

This Report compares the existing methods for solving the geodetic boundary value problem, i.e., the gravimetric determination of the geoid (traditional methods) or the physical surface of the Earth (modern methods). A unified treatment is attempted by deducing all these methods from one very general integral equation. Gravity reduction is discussed from the traditional and the modern points of view. The different kinds of free-air anomalies and free-air reductions, which are a controversial subject, are compared. Emphasis lies on the general aspects of the methods discussed.

153 - MORITZ H. - Green's function in physical geodesy and the computation of the external gravity field and the geodetic boundary value problem.
I.G.P.C. Columbus. AFCRL 65-522, Rep. n°49, 68 p.

"The present Report shows how Green's functions may be used for re-deriving known formulae and also for obtaining new results.

Formulae for the third boundary value problem for the sphere are developed and then specialized to Stokes's problem. As a limiting case, the boundary value problem for the plane is briefly considered. Then a formula for the variation of Green's function with the boundary surface is developed and applied to th problem of Molodenskij.

The main purpose of the present paper is to give formulae and, as an appendix, some estimates for the effect of topographic height on these computations. An application of Green's identities yields direct, but complicated, formulae for the effect of the disturbing potential T and the gravity anomaly Δg outside the Earth. A simpler solution for T is obtained through the use of a fictitious surface layer, a coating, on the Earth's physical surface. A third method, which seems to be optimal for practical computations, is a free-air reduction to sea level. The accurate performance of this reduction is a problem of downward continuation of Δg, which may be handled by iterative solution of a simpler integral equation. After this reduction, however, the conventional spherical formulas can be applied.

In addition, the paper presents connections between the determination of the external gravity field from surface data, which is related to the conventional boundary value problems of potential theory, and the determination of the Earth's physical surface itself, which is specifically geodetic boundary value problem.

154 - MORITZ H. - The gravimetric determination of the parameters of the Earth and its gravity field.
I.G.P.C. Columbus. AFCRL 65-655. Rep. n°51, 32 p.

"The present Report describes the determination of the geometric and gravimetric constants of the Earth from surface measurements. The starting point is a generalization of Stokes's formula to an arbitrary ellipsoid of reference, which is necessary because this formula, in its original form, imposes on the reference surface a condition that is not satisfied in practice.

The relation between gravity measurements and the determination of the physical constants of the Earth and its gravity field, such as its mass (the gravitational constant of the Earth), the potential at sea level, and the coefficients of the development of the external potential in terms of spherical harmonics., is developed. The theory of the mean Earth ellipsoid permits a rigorous definition of the Earth's equatorial radius, its flattening, and equatorial gravity.

Finally, additional related problems such as the effect of topography, accuracy aspects, and the application to satellite geodesy are briefly discussed.".

155 - MULLER I.I. - The geodetic applications of satellites.
I.G.P.C..Columbus. 1964, n°34, 68 p.

"This Report describes the methods of satellite geodesy for such geometrical and dynamical purposes as :

1) Determining the geocentric positions of observing stations when the orbit is known, and not known.

2) Improvement of assumed orbital elements with and without the simultaneous determination of the major semi-axis of the orbit and the period of the satellite, i.e., without assuming that the gravitational constant is fixed.

3) Determination and application of the various spherical harmonics coefficients.

Numerical results so far achieved are also give. A bibliography concludes this paaper".

156 - MURALEV Ja. G. - Formules intégrales...
sur une surface non de nivellée, voisine de l'ellipsoide de référence.
Izvest. Vys. Geod. Aerofot. 1964, n°5, p.66-74.

157 - ORLOVA E.M. - About accuracy obtainable in computing the influence of distant zones upon the quasigeoidal height by expanding the Stokes function in series of coefficients K and Q.
Trudy Cniigaik. Moscou. 1962, n°145, p.61-70. (russian text english abstract)

Using as an example, an anomalous field determined by functions of different complexity, a comparison is made between two methods of computing the influence of distant zones upon the quasigeoidal height. Both methods are based on an expansion of the stokes function in series of spherical functions with the aid of coefficients K and Q. A comparison of the results with the rigorous values has shown that the influence of distant zones can be determined from coefficients K at least several times as accurately as from coefficients Q. The convergence of K approximations is also much better. The advantage of using the coefficients K is revealed the clearer the more complex is the field of anomalies.

158 - PELLINEN L.P. - Influence of topographic masses upon the deduction of characteristics of Earth's gravitational field.
Trav. Inst. Geod. & Cart. Moscou. 1962, n°145, p.23-42. (russian text, english abstract)

This article is similar to the one published in Bul. Géod. 1962, n°63, p.57-66, already mentioned in Bul. Inf. Avril 1962, p.57.

159 - PELLINEN .P. - Expedient formulae for computation of Earth's gravitational field characteristics from gravity anomalies.
Bul. Géod. 1964, p.327-334.

"A transformation of the Molodenskij first-approximation formula for gravimetrical heights of quasigeoid permits to avoid computation of quantity which are largely dependent upon features of topography..."

160 - PELLINEN L.P, V.A. TARANOV & A.I. SABANOVA. - Calcul des hauteurs gravimétriques quasigéoidales et des déviations de la verticale sur CBM. "URAL - I"
Geod. Kart. 1964, n°12, p.9-13.

161 - PELLINEN L.P. - The averaging of anomalies and of G_1 - corrections in computing the characteristics of the gravity field by Molodenskij's formulae. Studia Geophys. & Geod. 1965, v.9, n°2, p.150-155.
(russian text, english abstract)

The formula for the mean value of sum of free air anomalies plus the G_1 - correction is obtained when the area in question is sufficiently large. It is shown that this sum is equal to that of the mean value of incomplete topographic anomalies and Bouguer corrections, when the Bouguer coefficient is selected optimally, plus the $\delta\bar{G}$ - correction (practically calculated with r_{min} - minimum distance between gravity points). Recommendations are given to calculate the K_{opt} $K(r_{oi})$ and $\delta\bar{G}$ values using an electronic computer by the method of statistical analysis of the heights and anomalies. It is noted that the $\delta\bar{G}$ corrections may be neglected as usual.

162 - PELLINEN L.P. - Use of the first - approximation formulae for calculation of the Earth's gravitational field characteristics. Trudy Cniigaik. 1965, n°157, p.85-99.

The methods for numerical calculation of the Earth's gravitational field characteristics are considered according to the Molodenskij's first approximation formulae.

In the cases when the mean values of gravity anomalies and G_1- corrections obtained for some areas can be used, it is recommended to determine the mean values of G_1 omitting the determination of their point values. When the radius of the averaging area is longer than 20 km it is practical to use, instead of G_1, more small and more stable G' - corrections. When free air anomalies are used, then the mean values of these corrections are close to the mean values of terrain reduction.

The method is considered for combined calculation of the mean values of Δg and G' from the data of the correlation - analysis of the gravity anomalies and the heights of the Earth's physical surface.

It is recommended to eliminate the constant part from the anomalies when the effect of the nearest areas on the quasigeoidal heights and plumb-line deflections is calculated.

In this case the formulae (46, 48, see text) are recommended for use to obtain the quasigeoidal heights and plumb-line deflections. Rough evaluation of the mean values of the G_2 and G_3 - corrections is made for large areas.

163 - PELLINEN L.P. - On some convenient transformations of Molodenskij's formula for quasigeoidal heights and plumb-line deflections. Studia Geophys. & Geod. 1965, v.9, n°2, p.124-128.
(russian text, english abstract)

"The influence of the constant part of the anomaly field on the quasigeoidal heights and plumb-line deflections is determined when Molodenskij's first approximation formulae are used. It is proved that this effect should not exceed 0.03 in plumb-line deflections, and therefore it may be neglected. It is recommended to eliminate the anomaly at the studied point from the anomaly field in advance, when calculating quasigeoidal heights and plumb-line deflections in the mountainous areas. Then the G - corrections can be determined. The effect of the constant part can be calculated from the given formulae."

164 - PICK M. - Einfluss eines der Systematischen Fehler bei der Geographischen. Studia Geophys. Geod. Prague, 1960, v.4, n°3, p.205-216.
(german text, russian abstract)

165 - PICK M. - Die allgemeine Stokessche Funktion und die von ihr abgeleiteten Funktionen. Trav. Geophys. Prague. 1961, n°126, p.11-14.
(german text, russian & czech abstracts)

166 - PICK M. - On determination of shape of Earth by using analytical continuation of function.
Studia Geophys. Geod. 1965, v.9, n°3, p.219-224.

167 - RAZINSKAS A.K. - Au sujet de l'étude détaillée du quasigéoide dans des conditions de plaine.
Trudy Akad. Nauk. Lit. SSR. 1962, B, 3(30) p.179-182.

168 - SHABANOVA A.J. - Calculation of Earth's gravitational field characteristics with the aid of the electronic digital computer "URAL-I" under utilization of gravity anomalies expansion in series of spherical functions.
Trudy Cniigaik. Moscou. 1962, n°145, p.77-82.
(russian text, english abstract)

During 1960 the following calculation programs were compiled and tested at the Central Scientific Research Institute of Geodesy, Aerial Surveying and Cartography:

1) A standard program for calculating spherical functions.

2) A program for computing the coefficients of a gravity anomalies expansion in series of spherical functions up to an order n = 16.

The calculations are carried out according to the slightly modified second method of Neumann.

3) A program for calculating, from an expansion available gravity anomalies, quasigeoidal heights and plumb-line deflections are made according to the formulae of Stokes and Vening Meinesz, respectively.

The described method of calculation is effective in all cases when the expansion available has been brought up to an order $n_o \leq 16$.

It is possible to adapt the given program for calculating the influence of distant zones upon quasigeoidal heights and plumb-line deflections, as well as for determining the characteristics of Earth's gravitational field in outer space.

The described set of calculation programs may be used for expanding any desired function of geographical coordinates in series of spherical harmonics, as well as for computing this function from a given expansion.

169 - STRANG van HESS G.L. - On the accuracy of the coefficients in a series of spherical harmonics.
I.G.P.C. Columbus. 1964, Rep. n°40, 24 p.

"The accuracy of the coefficients of a development in series of spherical harmonics is investigated. Usually the coefficients are determined by numerical integration. The mean value of several surface elements is estimated. Representation error and integration error are distinguished. Topography in spherical harmonics and gravity anomalies in spherical harmonics are considered".

170 - TENGSTROM E. - Symposium on the determination of the figure of the Earth held in Prague, 5-10 October 1964.
- C.R. of the Meeting.
- Rèsolutions adopted.
Bul. Géod. 1965, n°75, p.5-12.

171 - TIRON I.M. - Etude de l'exactitude des formules de Stokes et de Vening Meinesz.
Trudy Cniigaik. 1960, n°139, p.21-45.

172 - TIRON M. & S. STRUTU - Some problems regarding the way of sölving Molodenskij's integral equation for the Earth considered as a plane.
Studia Geophys. & Geod. 1965. v.9, n°2, p.137-143.

"... In our opinion, this is where Molodenskij's theory is superior to all those proposed previously, Stokes's included.

The purpose of this work is not to emphasize the superiority of Molodenskij's theory, which is generally known, but to establish relations which permit an approximation as accurate as possible of the fundamental integral equation in the near vicinity of the studied point, and also to give the formulae necessary for the construction of the calculating templates...".

173 - ZAGREBIN D.V. - Sur la formule de Stokes dans le cas d'une surface de niveau ellipsoidale.
Nauk. Zap. Louvosk. Polit. Inst. 1962, n°85, p.68-82.

174 - ZONGOLOVIC D. & L.P. PELLINEN - Certains aspects de la solution du problème fondamental de géodésie supérieure.
from "The use of artificial satellites for geodesy". 1963, p.341-345.
Proc. of the 1st Int. Symp. Washington (26-28 April 1962).

The size and shape of the Earth can be determined from either measurements on the surface of the Earth or measurements to cosmic objects. The data available now from either source are not sufficient to give a unique solution, and there are differences of opinion on their interpretation. The problem today is to find methods that will make the best use of the date. The importance of combining the terrestrial and extraterrestrial data is emphasized.

175 - ZIELINSKI J. - Contributions of satellite method to the gravimetrical and geometrical measurements.
Bul. Géod. 1965, n°76, p.135-144.

"It seems reasonable to introduce a hierarchy of observations, according to the possibility of each of the methods (geometrica, gravimetric or satellite method). First of all, taking into account the obsarvations of the artificial satellites we can determine the general geoid's shape (the quasigeoid). The response successive terms of the series expansion is conditionned by the actual possibilities of the technics, observation and calculation. For instance, at present, the J_3 value is known with such a high accuracy, that it can be considered as a constant in adjusting the gravimetrical measurements. Assuming that special "gravimetrical" satellites will be launched it can be expected that we will be able to determine with a sufficient accuracy the harmonics of the 5th order (functions of ϕ as well as of λ). Introducing those values as constants into the computations, we can determine the coefficients of terms of higher orders, even from incomplete gravimetrical data... Having the shape of surface determined, we are able to find its size by the determination of the ellipsoid's parameters..."

C) RESULTATS

176 - ARNOLD K. - The European datum.
Boll da Univ. do Parana, Geod. 1962, n°4, 25 p.

"An historical outline of the determinations of the European datum during last fifteen years is followed by an extensive description of a new investigation of the gravimetric deflection at the Helmert-Tower of the Geod. Inst. at Potsdam which is generally chosen as initial point of the European Geodetic System.

All available up to data gravity measurements were employed in order to obtain the deflection of the vertical at the Helmert-Tower, Potsdam, expressed as a function of free air anomalies".

177 - ARNOLD K. - Die Freiluftanomalien im Europäischen Bereich.
Deutsche Geod. Inst. Potsdam. 1964, n°25, 56 S.

Tableau des anomalies moyennes à l'air libre, calculées à partir de la formule d'Helmert (1901) pour des carreaux de 1° x 1°, 1° x 30', 30' x 20', 20' x 12' ou 10' x 6', suivant les régions.

La région considérée est comprise entre les latitudes 72° N et 40° S et les longitudes 12° W.G. et 36° E.G.

178 - ARNOLD K, D. SCHOEPS & L. STANGE - Numerical investigations on the correction term for the free air anomalies according to the new theory of the dynamical geodesy.
Numerische Unterschungen zum gravimetrischen Zusatzglied
Gerlands Beitr. Geophys.
C.R. Studia Geophys. & Geod. 1965, v.9, n°2. p.160 (M. Pick)

"A new practicable scheme for the numerical determination of the correction term for the free air anomalies according to the so-called "New theory" of gravimetrical geodesy has been developed in order to obtain the potential and the deflections on the surface of the Earth. The scheme consists in a square net for which the heights are to be interpolated from maps, to handle them further by an electronic computer yielding the wanted correction term. The first applications were executed for the Harzmountains area".

179 - ARONOV V.I. - Sur la réduction des anomalies de pesanteur dans les régions montagneuses.
Sbor. Geo. Iziceskaja Razvedka. 1963, n°14, p.80-91.

180 - AVDULOV M.V. - Evaluation de la précision des cartes gravimétriques. Sbor. Prikl. Geophys. 1963, n°36, p.167-180.

181 - BOKUN J. - Remarques sur l'exploitation des données gravimétriques pour l'élaboration d'un réseau de triangulation. Bull. Géod. 1963, n°68, p.145-148.

182 - CORON S. & D. KOTLARZ - Anomalies moyennes à l'air libre et anomalies moyennes de Bouguer. Bull. Géod. 1962, n°69, p.239-248.

Quelques arguments pour l'emploi des anomalies de Bouguer pour le calcul indirect des anomalies moyennes à l'air libre.

183 - DRAHEIM H. - Das Geoidprofil Giessen - St-Gotthard - Brissago. Deutsche Geod. Kommiss. München. 1963, Reihe B, n°104, 13 S.

184 - DURBIN W.P. JR. - Comparison of crustal parameters with geoid undulations. Report presented to the 13th G.A. of the UGGI, Berkeley, 1963, 7 p. 16 fig.

" The gravimetrically determined geoid bears a marked resemblance to configurations of the crust in the U.S. Since several correlations between the crust and the force of gravity have already been demonstrated, an investigation into geoid crust relationships is in progress at ACIC. Geoid profiles across the U.S. have been compared to the crust, geologic structures, and various gravity expressions. The preliminary results of the investigation are presented in this paper. For those instances in which a distinct relationship exists, a concept of regional isostatic compensation can be acquired from computed geoids. Conversely, explosion seismic interpretations can be transformed from crustal thicknesses to geoid undulations."

185 - DURBIN W.P. - Geophysical correlations A.C.I.C. Saint-Louis. July 1965, 11 p.

" Where gravity data is insufficient for geodetic purposes and actual observations are not practical, non-conventional methods must be used. This paper proposes several geophysical approaches toward partial, and magnetic information in a gravimetrically unsurveyed area will provide an approximate anomaly which can either stand alone or be used in conjunction with statistical methods."

186 - FISCHER I. - Parallax of the moon in terms of a World geodetic system.
Astr. J. Washington. 1962, n°6, p.373-378.
U.G.G.I. Army Map Service, Berkeley. 1963, 36 p.

"The recent geodetic connection between Europe and South Africa along the 30th meridian east makes it possible to reinterpret the Greenwich and Cape observations of the lunar crater Mösting A made by Christie and Gill in the years 1905 to 1910. Cromelin's result for the correction of Hansen's value for mean parallax of the moon is expressed as a mean lunar distance in kilometers, which is compared with various results from dynamical theory, with O'Keefe and Anderson's determination from occulations, and with radar measurements. The Astrogeodetic World Datum serves as common reference system. As a by-product, the contradictions in the literature quoting Hansen's value for mean parallax are cleared up."

187 - GLENNIE E.A. - Mapping the model Earth.
Typewritten text presented to the Int. Grav. Comm. Paris, Sept.1962.5p.

188 - GOVOROVA L.A. - On errors in interpolation of gravity anomaly and on accuracy of gravimetric deviations of vertical.
Trudy Cniigaik. 1960, n°139, p.77-81.

189 - GROTEN E. & H. MORITZ - On the accuracy of geoid heights and deflections of the vertical. I.G.P.C. Columbus.1964, Rep. n°38, 36 p.

" The accuracy of the geoid undulations and the deflections of the vertical, obtainable by means of different idealized gravity nets, is investigated. The following cases are assumed : one gravity station or one gravity profile in every 1° x 1°, 2° x 2°, 5° x 5°, and 10° x 10° blocks".

190 - GROTEN E. - On free air anomalies estimated from different models
I.G.P.C. Columbus. 1964, Rep. n°42, 22 p.

"From mean free air anomalies of 124, 5° x 5° squares as observed on the northern hemisphere, the accuracy of model anomalies for different models is tested. Only mean elevations are used in the evaluation of topographic effects."

191 - GROTEN E. - On gravity reductions.
I.G.P.C. Columbus. 1964, Rep. n°44, 24 p.

"Bouguer's and isostatic methods for the evaluation of gravity anomalies on the Earth's physical surface are discussed.

Errors involved in the reduction of gravity down to sea level are compared for condensation, isostatic, and Rudzki's anomalies.

The connections between gravity anomalies at the Earth's surface and on the geoid are discussed".

192 - GROTEN E. - On gravity prediction using mean anomalies
I.G.P.C. Columbus. AFCRL 65-520. Rep. n°47, 16 p.,1965.

" The efficiency of prediction of mean gravity anomalies for areas of 1° x 1°, 2° x 2°, 5° x 5°, and 10° x 10° from known mean anomalies is studied; The least standard prediction error is found to be about 1/2 $(var)_s{}^{1/2}$ where var_s is the variance of mean anomalies. The influence of the nearest neighbors is prevailing. The prediction from many known anomalies is about the same as from only a few known values. "

193 - HEISKANEN W.A. - Activity of the Columbus geodetic group in physical geodesy since 1960.
Publ. Isos. Inst. 1964, n°41.
(Ann. Acad. Aci. Fenn. Ser. A, III, Geol. Geogr. n°72, 26 p.)

1) Purpose of the physical geodesy
2) Historic of the studies carried out on the figure of the Earth, since the XIX century.
3) Need to compute all data in the same world gravimetric system.
4) Summary of the different methods :
   - to extrapolate anomalies in unsurveyed areas
   - to get anomalies at different altitudes.

194 - KARTVELISVILI K.M. - Sur la précision du calcul des anomalies de pesanteur dans les régions montagneuses.
Geophys. Inst. Sak. SSR. Mec. Akad. 1963, n°21, p.221-236.

195 - KIVIOJA L. - Development of gravity Bouguer anomalies of State of Ohio and the isostatic anomalies in North Atlantic in Fourier Series.
I.G.P.C. Columbus. 1962, Rep. n°22, 13 p.

"Gravity anomalies were developed in Fourier series in two test areas : 2° x 3° area in the State of Ohio, and 10° x 35° area in the Atlantic. Based on these developments mean gravity anomalies were estimated for 10° x 10° and 1° x 1° squares.
For comparison also another method was used in which the original values were plotted on a transparent sheet placed over a topographic or bathymetric map, isanomaly coutour lines were drawn based

only on the original anomaly values and the topography, and the mean gravity anomalies were estimated for the same squares as in Fourier series method. The result is that this second manual method has smaller standard errors than the Fourier series method, and that this kind of extrapolation even between two anomaly profiles should not be extended further than about 4° from a profile.

The topographic method is more accuracte, faster, and it can better use all available material regardless of whether the gravity points are along the profiles or scattered over a large area. The combination of analytic and topographic methods will obviously bring the best results. "

Tableau des anomalies moyennes de Bouguer par carreau 0°5 x 0°5. (Lat. 39° - 41°5, long. 81° W.G. - 84°5).

196 - KIVIOJA L. - Effect of topographic masses and their isostatic compensation on the mean free air gravity anomalies of 5° x 5° surface elements.
I.G.P.C. Columbus. 1963, Rep. n°28, 135 p.

" The effect of topography and its isostatic compensation for all 5° x 5° surface elements has been calculated, using the T = 30 km Airy-Heiskanen local isostatic system, and the available 5° x 5° mean free air anomalies based on measurements.

Measured 5° x 5° mean free air anomalies on the Atlantic Ocean, on the North American Continent, and at its shore, were the test areas used for the adjustments.

In the test areas, these calculated 5° x 5° mean free air anomalies would represent the measured 5° x 5° mean free air anomalies so that the standard errors on the oceans are about ± 6 mGal, on continents and at the coasts about ± 11 mGal. The calculated anomalies are developed into spherical harmonics up to the 8th order.

Also, 5° x 5° mean free air anomalies for the entire Earth are given consisting of 5° x 5° mean free air anomalies based on measurements wherever available as given by Uotila, and the remaining 5° x 5° elements have anomalies extrapolated from known and calculated anomalies.

Mean elevations for all 5° x 5° elements were estimated, and are given in meters.

The mean effect of topography and its isostatic compensation of the Hayford zones 18-1 is given in the Airy-Heiskanen, T = 30 km isostatic system for every 5° x 5° surface element. "

197 - MADER K. - Topographische berechnete partielle Geoidhebungen.
Osterr. Z. VermessWes. 1960, n°21, p.14-32.

L'élévation partielle du géoide due aux masses de la montagne est calculée en tenant compte de la topographie jusqu'à une distance de 30 km.

198 - MONIN I.F. - Formule de la répartition de la pesanteur pour la région européenne de l'U.R.S.S.
Nauk. Zap. Louvosk. Polit. Inst. 1962, n°85, p.94-99.

199 - MORITZ H. - Accuracy of mean gravity anomalies obtained from point and profile measurements.
I.G.P.C. Columbus, 1963, Rep. n°29, 23 p.
Publ. Isos. Inst. Helsinki. n°45.
(Ann. Acad. Sci. Fenn. Ser. A, III. Geol. Geogr. n°76)

"Mean gravity anomalies for a certain rectangular area (block) can be computed from observed point or profile gravity values using interpolation method (e.g., representation or least squares interpolation). If the covariance function of gravity anomalies is known and the interpolation method is specified, then we can compute the accuracy (standard error and error covariance) of the mean anomalies without any additional numerical data.

The formulas for this purpose which were partly published earlier are evaluated numerically for different sizes of blocks and different interpolation methods using an estimate for the covariance function given by W.M. Kaula in 1959. The integrals involved are approximated by summations performed by high-speed computer.

The results show the accuracies obtainable with a given observational material, by different interpolation methods. They are useful for numerical studies on the error propagation in the computation of gravimetric quantities (e.g., geoid undulations).

200 - MUELLER I.I, O. HOWAY & R.B. KING - Geodetic experiment by means of a torsion balance.
I.G.P.C. Columbus. 1963, n°30, 71 p.

Le travail expérimental décrit dans cet ouvrage est résumé en partie dans "Interpolation of deflections of the vertical by means of a torsion balance", par Mueller I.I. (voir p.70)

201 - RAMSAYER K. - Proposal for a gravity map for hilly and medium mountainous areas without the calculation of the terrain correction.
Text presented to Berkeley, Aug. 1963. 5 p. (english text)
German text published later :
"Vorschlag für eine Schwerekarte für Hügelland und Mittelgebirge ohne Berechnung der topographischen Reduktion".
D.G.K. München. 1963, Reihe B, n°104.

202 - RAPP R. - Correlation coefficients and their use in the prediction of mean anomalies.
I.G.P.C. Columbus. 1962. n°20, 38 p.

"The correlation of point anomalies with the separation between them has been investigated by means of the correlation coefficient $r_{o,s}$. Correlations curves for free air anomalies have been established in Ohio and Finland, and for Bouguer anomalies in Ohio alone. The individual curves showed a wide variation but the curve obtained by meaning the values of a particular $r_{o,s}$, from the individual curves gave a reasonably smooth curve that turned negative at 107 km in Ohio and 147 in Finland.

Various equations were applied to these mean curves in order to find an analytical representation.

Tests of the prediction formulas for free air anomalies were carried out using circles of 20 km and 40 km in radius. An accuracy analysis was made by comparing the predicted mean anomaly with a "true" mean anomaly. Based on 14 points in Ohio a mean error of one prediction was $\pm$ 4 mGal and on 15 points in Finland was $\pm$ 3 mGal (for R = 40 km). In both cases this was an increase in the accuracy of mean anomaly above that if the mean anomaly was simply taken as the point anomaly at the center of the circle... It seems valid to assume that data from one area can be applied to another area as long as the areas are similar (i.e, in this case, relatively flat)... It is possible that also in areas of rugged topography the data of an area A can be applied in another area B, but then the elevation correlation (of the free air anomalies) has to be considered. "

203 - RAPP R. - Statistical analysis of gravity anomalies and elevations by long profiles and by areas.
Publ. Isos. Inst. Helsinki. 1963, n°48, 11 p.
(Ann. Acad. Sci. Fenn. Ser. A, III, Geol. Geogr. n°79)

Statistical analysis of gravity anomalies is needed for such purposes as the prediction, extrapolation and interpolation of such anomalies.

The equations used here are the usual statistical formulae except for certain modification needed for this particular problem. The data examined was that of 5' x 5' means in terms of strips in either latitude and/or longitude extent. After the analysis of each strip in a certain area, means were taken to give a value of a parameter for a specific area.

The data used in the computations consisted of 5' x 5' means of elevations, Bouguer anomalies and free air anomalies. The latter were computed from the Bouguer anomaly and elevation data using the relation : $\Delta g_{FA} + \Delta g_{BA} + 1119h$ (for land areas).

In using the data analyses were carried out in 2 ways : one was to consider some square areas, and to compute the statistical functions in an E - W and S - N direction obtaining means for each direction. In this method the area was searched for a strip that was at least 14 blocks long (about 102 km in an E - W direction). The next investigation was that of long profiles across the U.S. (Average length of a profile was 4 000 km)..."

204 - RAPP R. - The prediction of point and mean gravity anomalies through the use of a digital computer.
I.G.P.C. Columbus. 1964, n°43, 177 p.

"The lease squares predictions of point anomalies has been carried out in two distinct areas of the United States. The statistical quantities needed for their use in the prediction equations of Moritz were computed using some 32.000, 5' x 5' mean anomalies.

The meam anomalies and their accuracies in these 2 areas were predicted in 5' x 5' and 30' x 30' squares. Computer programs for the IBM 7094 were used in computations".

205 - RAZINSKAS A.K. - Problèmes géodésiques et gravimétriques en Lithuanie.
Trudy Geod. Sekcii. Akad. Nauk. Lit. SSR. 1963, n°1, p.15-20.

206 - SCHEIBE D.M. - Density and distribution of observations in determining mean gravity anomalies.
ACIC. Saint-Louis. April 1965, 14 p.

" This study develops a method of determining the minimum number of observations required for valid 1° x 1° mean gravity anomalies. An area with dense gravity coverage is used for the test computations. The mean gravity anomalies are determined by least squares solutions utilizing all the available observations. Through successive computations mean gravity anomalies for the area are then determined with statistically reduced numbers of observations to establish the effect on accuracy. Mean gravity anomalies are also computed from stations along selected geological profiles. In every case, those considering geological factors are more reliable for minimum numbers of observations required. "

207 - STRANGE W.E. & G.P. WOOLARD - The use of geologic and geophysical parameters in the evaluation, interpolation, and prediction of gravity. ACIC. USAF.
ACIC. USAF. Hawaii Inst. of Geophys. Nov. 1964, HIG-64-17. 142 p.

Final Report : Part I

"The correlations developed in this first Report represent input information which can be used, in addition to knowledge of the spatial relations of points at which gravity has been observed, to carry out evaluation, interpolation, and prediction of gravity. By use of these correlations, considerable improvement can be realized."

207 bis - STRANGE W.E. & G.P. WOOLARD - The prediction of gravity in the United States utilizing geologic and geophysical parameters. ACIC. USAF. Hawaii Inst. of Geophys. Nov. 1964, HIG-64-18. 64 p.

Final Report : Part II

Tables of the average elevation, gravity anomalies for 1° x 1° or 2° x 2° square.

"Correlations developed between gravity anomalies and geologic and geophysical parameters were utilized for the prediction of gravity within the continental United States. The 2° x 2° mean free air anomalies predicted using the geologic geophysical correlations had a mean error without regard to sign of 7 mGal and a r.m.s. error of 8.8 mGal. The predicted anomalies for 1° x 1° squares had a mean error without regard to sign of 10 mGal and a r.m.s. error of 12.9 mGal.

The results obtained here indicate that a significant improvement in the accuracy of prediction of free air anomalies can be obtained if geologic and geophysical information is used to control the prediction".

208 - STRAUB G. - Breitenbestimmungen und Geoidstudien im Meridian des St. Gotthard. Deutsche Geod. Komm. München. 1963, Reihe C, n° 65, 101 S.

209 - TARANOV V.A. - Deducing relation between gravity anomaly and elevation to obtain mean gravimetric data for large areas. Trudy Cniigaik. Moscou. 1962, n°145, p.71-76. (russian text, english abstract)

"A research method and the results of an investigation are described. This method has been carried out at the Central Scientific Research Institute of Geodesy, Aerial Surveying and Cartography with the view to establishing relations between gravity anomalies, and elevation and depth in order to obtain mean gravimetric data for squares 10 x 10° ; a comparison is also made with similar investigations of Zongolovic, Jeffreys, and Kaula.

A relation was obtained showing the influence of elevation and depth upon two kinds of gravity anomalies (free air anomalies - Δg, and free air anomalies with terrain correction - $\Delta g'$) at four successive stages of averaging the anomalies :

1) For squares 1 x 1°,
2) For squares 2 x 2°,
3) For squares 4 x 4°,
4) For squares 10 x 10°.

For every stage the following coefficients were obtained characterizing the dependence of anomalies Δg and $\Delta g'$ upon elevation (k_i and k'_i) and depth (b_i) :

successive stages of averaging the anomalies	size for squares	mGal/m k_i	mGal/m b_i	mGal/m k_i'
I stage	1° x 1°	+0,094±0,004	(-0,0686)	+0,111±0,003
II stage	2° x 2°	+0,054±0,004	-0,019±0,003	+0,059±0;002
III stage	4° x 4°	+0,026±0,004	-0,010±0,002	+0,032±0;002
IV stage	10° x 10°	+0,0124± 0,0015	-0,010±0,002	+0,0165

Within the areas 1 x 1° the obtained coefficient showing the relation existing between anomalies and elevation approaches the coefficient of Bouguer reduction. As the area being averaged increase, the influence of terrain upon gravity anomalies becomes weaker but does not completely disappear, and for areas 10 x 10° makes up about 10-15 % of the Bouguer coefficient."

210 - TENGSTROM E. - <u>Interpolation of deflections by means of gravimetric data</u>. Studia Geophys. & Geod. 1965, v.9, n°2, p.211-213.

211 - TENGSTROM E. - <u>Report on the use of recent satellite results for improving absolute deflections and geoid heights</u>. Studia Geophys. & Geod. 1965, v.9, n°2, p.173-175.

A map gives geoidal free air anomalies from recent satellite observations including 4th order terms, (flattening 1/297) and the 5° x 5° free air anomalies from surface measurements and international gravity formula.

212 - TENGSTROM E. - Test area in the West-Alps.
Studia Geophys. & Geod. 1965, v.9, n°2, p.162-165.

General view on the test work for computing differences of gravimetrical deflections of the vertical in or near rough topography by different methods to the compared with astrogeodetically determined deflections.

213 - TENGSTROM E. - The present state of the test work in the West-Alps.
Typewritten text. Bul. Inf. Juin 1965, n°13, 5 p. + 5 fig.

214 - UOTILA U.A. - Corrections to gravity formula from direct observations and anomalies expressed in lower degree spherical harmonics.
I.G.P.C. Columbus. 1962, Rep. n°23, 26 p.

"The author had at his disposal 11,586 1° x 1° mean free air anomalies and 8,172 1° x 1° mean isostatic anomalies which were determined from surface gravity observations. Using the least squares solution, he computed corrections to the International Gravity Formula, and he also computed the theoretical gravity formula for the triaxial ellipsoid. He determined the coefficients for lower degree spherical harmonics. These were computed two different ways :

1) Using surface gravity anomalies alone,

2) Combining surface gravity information and zonal harmonics computed from satellite data.

Departures from the International Gravity Formula, and the contributions of the lower degree spherical harmonics to the undulation of the geoid are given in map form. All computations are referred to Potsdam System, g = 981,274 00 Gals. The solution computed from free air anomalies does not agree with values computed from satellite data as well as the result from isostatic anomalies does. Above isostatic anomaly solution gives 18° West from Greenwich for the location of major axis of the equator. Izsak's value is 33° West and Kozai's 37° West from satellite observations. These values do not agree with the solution above from direct gravity observations...

The results suggest very strongly that the Northern hemisphere is different from the Southern hemisphere."

215 - UOTILA U.A. - Gravity anomalies for a mathematical model of the Earth.
Publ. Isos. Inst. Helsinki. 1964, n°43, 14 p.
(Ann. Acad. Sci. Fenn. Ser. A, III, Geol. Geogr. n°74)

" The author used 5° x 5° blocks bordered by the meridians and parallels.

He has also computed coefficients for the squares of the elevations. Using Jung's derivations, the author computed free air and isostatic anomaliss for the Airy-Heiskanen T = 30 km model using densities 2.67 g/cm^3, and giving the water a den sity of 1.027 g/cm^3. The corresponding model free air anomalies are given. The coefficients for spherical harmonics that express the model free air anomaly field in milliGals are given.

The model is a continous mathematical surface and is determinable for any point of the surface. It also shows the correlation between topography and gravity anomalies.

The author suggests that the computed model anomalies should be removed first from observed anomalies and studies should be done with the residuals. Using these residuals, the correction to the flattening as well as the equatorial value could be computed."

216 - VYSKOCIL V. - Some remarks on the accuracy of interpolation of gravity anomalies.
{Studia Geophys. & Geod. 1963, n°7, 228 p.
{Travaux Inst. Geophys. Acad. Tchecosl. Sci. 1963, n°180. }
Summary in : Studia Geophys. & Geod. 1965, v.9, n°2, p.161-162.

"This paper presents the main features of the method estimating the accuracy of interpolated values of gravity anomalies. Its main idea was formulated by Molodenskij and developed mainly by Grushinsky. It has been completed by the author in the problem of distinguishing the effect of anomaly errors at the points of observation from the effect of errors of the interpolation itself and by analyzing their effect in the field of interpolation".

217 - WARD M.A. - On detecting changes in the Earth's radius.
Geophys. J. 1963, v.8, n°2, p.217-225.

"Estimates of the Earth's radius in the geological past can be made from paleomagnetic evidence. A method appropriate to the spherical environment of the data for dealing with this problem is given, which is applied to Devonian , Permian and Triassic data from Europe and Siberia, yielding estimated radii for these periods of 1°12, 0°94 and 0°99 times the present radius respectively. These estimates are not considered to be significantly different from the present radius. "

218 - ZEMLAJAKOV V.N. - Possibilité d'augmenter la précision du calcul des masses topographiques dans les régions montagneuses.
Inf. Sbor. Primorsk. Geolo. Uprav. Glavnogo. Geolo. Ohrany. Sovete Ministrov. RSFSR. 1963, n°4, p.69-74.

219 - ZONGOLOVIC I.D. - Revue des résultats de la détermination des paramètres du champ de pesanteur terrestre obtenus par les observations des satellites artificiels.
Publ. Spe. 1962, p.25-32.

DEVIATIONS de la VERTICALE et ANOMALIES de la PESANTEUR

21* - ARNOLD K. - The deflections of the vertical expressed in free air anomalies and their relationship to the gravimetric-isostatic deflections. Bull. Géod. 1962, n°65, p.259-264.

" The most important problems both in geophysics and geodesy all can be solved advantageously by isostatic anomalies. However, because the computation of isostatic reductions is a laborious work it seems adequate to recommend also the use of the Bouguer anomalies and the free-air anomalies. As far as the widely spread use of a special reduction method is concerned, this method should not involve any kind of the so-called "geologic-correction" and no correction with regard to the deviation of the real free-air gravity gradient from the normal value 0,3086 should be taken into consideration. Otherwise some shortcomings will be introduced into geodetic applications of the anomalies. At last no correction for the curvature of the plumb-line should be added".

22 - ARNOLD K. - The Helmert-Tower, a zero - order - point. Studia Geophys. & Geod. v.9, n°2, p.171-172.

Les différents résultats obtenus pour les déviations de la verticale au point suivant (Tour d'Helmert Geod. Inst. Potsdam) sont très concordants malgré les méthodes utilisées :

- Méthode géométrique (Wolf, système européen)
- Méthode gravimétrique (Arnold)
- Observations des satellites (Kaula)

Ce point est donc un des très rares points dont la position absolue est connue à mieux que ± 15 m (et la déviation à mieux que 0"5).

23 - ARONOV V.I. - Déviations de la verticale obtenues à partir des observations Δg dans une région montagneuse. Izvest. Vys. Zav. Geod. Aerofot. 1964, n°6, p.65-71. (texte russe)

24 - BURSA M. - Bestimmung des Interpolationsfehlers der astronomischen geodätischen Lotabweichungen im westlichen Teil Böhmens. Inst. Geoph. Acad. Tch. Sci. 1963, t.X, n°162, p.11-24. (texte russe, résumés russe, tchéque et allemand)

* Ces articles font suite à ceux indiqués dans le Bulletin d'Information (Avril 1962) p.63.

"1) Im Gebiet σ mit einer Dichte astronomisch-geodätischer Punkte 1. Ordnung ca. von 1700 km^2 pro Punkt, beträgt der mittlere Interpolationsfehler der astronomisch-geodätischen Lotabweichungen ungefähr ± 2,5".

2) Bei einer Anwendung der ausführlichen gravimetrischen Karte mit einer Dichte der Schwerepunkte von 5 km^2 pro Punkt, können mittels der Methode nach Molodenskij die Komponenten der astronomisch-geodätischen Lotabweichungen mit der Genauigkeit einer Zehntel Sekunden berechnet werden. Im angeführten Falle ergab es sich : $m(\xi, \eta) = \pm 0{,}26''$ (System I) und $\pm 0{,}48''$ (System II).

3) Im Falle, dass ausführliche gravimetrische Karten und eine ausreichende Anzahl astronomisch-geodätischer Punkte I. Ordung vorliegen, ist die Errichtung astronomisch-geodätischer Punkte II. und niederer Ordnung, mit einer Messgenauigkeit φ, λ ± 0,5" und kleiner, allgemein nicht begründet, obwohl dies in manchen Ländern bisher noch geschieht.

4) In astronomisch-geodätischen Grundnetzen sind, "astronomisch-geodätische Fundamentalpunkte" mit der realen Bestimmungsgenauigkeit der astronomisch-geodätischen Lotabweichungen ungefär von ± 0,1" zu errichten. Dann können an Hand ausführlicher gravimetrischen Karten die Komponenten der astronomisch-geodätischen Lotabweichungen mit der Genauigkeit einiger Zehntel Sekunden interpoliert werden".

25 - BUZUK V.V. - <u>Principe de la séparation des déviations absolues de la verticale (gravimétriques) en locales et générales (planétaires)</u>. Trudy Novosibirsk. Inst. Inzenerov Geod. Aerofot. Kartogr. 1964, n°18, p.9-13.

26 - CAMPBELL A.C. - <u>Rigourous solution for the determination of the deflection of the vertical for the center area.</u> Bull. Géod. 1963, n°69, p.281-292.

"In the evaluation of the center area for the determination of gravimetric deflections, the Stokes function approaches infinity as the radial distance approaches zero. This condition has tended to frustrate attempts of precise determination for the deflection effect for the center area. Over the years, a number of empirical solutions have been devised to approximate the effect of the center area. The solution devised by Rice is one of the better ones, and it is very easy to use . Rice's solution is used for comparison with the rigorous solution in this article.

The rigorous solution is given for both circular and square development. Since the evaluation of a square area is more difficult to directly compare for the same area where an inscribed circle is used to include the area, only one comparison will be given between the rigorous solution using squares compared to Rice's solution. The rigorous solution employing radial lines and circles is compared in four separate problems.

27 - EREMEEV V.F. & M.J. JURKINA - Remarks a propos of a short communication of I.F. Monin "on the calculation of the plumb-line deflection". Trudy Cniigaik. 1965, n°157, p.135-138.

"It is demonstrated that the method proposed by I.F. Monin is not applicable to practical use. It is further shown that the first and the second methods of Molodenskij of numerical solution of his integral equation for density of a single layer substituted for the attraction of anomalous masses in the interior of the Earth promptly lead to a higher accuracy than that of the Stokes's theory applied to the real Earth. The results of calculations carried out in accordance with the first of these methods are also much more accurate as compared with the Stokes's approximation, and are in fact near the correct values".

28 - FISCHER Irène - Gravimetric Interpolation of deflections of the vertical by electronic computer.

Army Map Service. Washington, 1965, p.1-14.

"The Army Map Service is in the process of revising its chart of geoidal contours in North America. We plan to use a closely spaced meridional profile through the middle of the country as a backbone for the construction...

... Description of the procedure."

29 - KOBOLD F. & E. HUNZIKER - Communication sur la courbure de la verticale. Bull. Géod. 1962, n°65, p.265-267.

" Le but de cette communication est de donner une idée sur l'ordre de grandeur de la courbure de la verticale sur plusieurs points en haute montagne (13 points de profil du St-Gothard)... La méthode consiste à calculer les déviations de la verticale pour la station d'observation à la surface de la Terre et pour le point correspondant au niveau de la mer. Ce calcul ne dépend pas du tout du choix d'un ellipsoide de référence mais se base uniquement sur l'influence des masses visibles."

30 - KROTKOVA E.I. - A template for the determination of influence of neighbouring zones on the quasigeoidal height and plumb-line deflection for use in computation of the influence of distant zones from the Molodenskij's formulae.
Trudy Cniigaik. 1965, n°157, p.109-115.

31 - MEIER H.K. - Uber die Berechnung von Lotabweichungen für Aufpunkte im Hochgebirge.
Sammelband Dissertationen, Teil II. Munich. 1959.
Deutsche Geod. Komm. Reihe C, H. 26, 29 S.

32 - MONIN I.F. - On the calculation of the plumb-line deflection.
Astr. J. 1963, v.XL, n°1, p.179-183.

33 - MUELLER I.I. - Interpolation of deflections of the vertical by means of a torsion balance.
Publ. Isos. Inst. 1964, n°47, 6 p.
(Ann. Acad. Sci. Fenn. Ser. A, III, Geol. Geogr. n°78.)

"Use of the torsion balance for interpolation of vertical deflections between astronomical stations, with satisfactory results, this method can be useful in cases where gravimetric interpolation is not feasible."

34 - ORLOVA E.M. - Tables destined for use in accounting for non-linearity of the influence of heights of topographical masses on plumb-lines deflections.
Trudy Cniigaik. 1965, n°157, p.101-108.

35 - PELLINEN L.P. - Errors in computation of plumb-line deviation during uniform gravimetric survey.
Trudy Cniigaik. 1960, n°139, p.83-87.

36 - PELLINEN L.P. & L.A. GOVOROVA - An estimate of accuracy of astro-gravimetric levelling in the USSR.
Trudy Cniigaik. 1962, n°145, p.43-59. (russian text, english abstract)

"The accuracy of computing gravimetrical and astro-geodetic plumb-line deflections at points of astro-gravimetric levelling in the USSR is investigated. Use has been made of gravimetric extension surveys carried out by the Establishment N7, as well as of detailed surveys conducted by geophysical organizations from 1947 to 1957, and of experimental work of 1960.

It is shown that variations of discrepancies between astro-geodetic and gravimetric plumb-line deflections (residual deflections) are due chiefly to errors in astronomical coordinates. The accidental and systematic parts of their effect upon the plumb-line deflection are $\pm$ 0".35 and $\pm$ 0".45, respectively, and both are of the same magnitude for deflection components along the meridian and the prime vertical. According to data of extension surveys carried out by the Establishment N7, the gravimetrical plumb-line deflections are determined with an error of $\pm$ 0".3. A new scheme of extension survey is recommended for those regions where detailed geophysical surveys are available".

37 - PICK M. - Deflections of the vertical in the high Tatras.
Studia Geophys. & Geod. 1965, v.9, n°2, p.156-159.

On 5 points in the High Tatras the deflections of the vertical (ξ_g , η_g) and the height of the quasigeoid (ζ_g) were determined from the gravimetric data using Molodenskij's method and the results were compared with astronomic-geodetic values (ξ_A, η_A, ζ_A) and with values (ξ_T, η_T, ζ_T) computed from the trigonometric net. The correspondence of the gravimetric and astronomic system is good, the components of the trigonometric deflections of the vertical parallel to the mountain chain are probably charged with a systematic error, depending on the height above sea level".

38 -SCHMIDT A. - La déviation absolue de la verticale (II)
Rev. Hydrogr. Int. Monaco. Jan. 1965, v.XLII, n°1, p.107-130.

Définition - vue d'ensemble.

39 - STRANG van HESS G.L. - Deflections of the vertical caused by topographic masses in the nearest neighborhood of the point.
I.G.P.C. Columbus. AFCRL. 1964, Rep. n°39, 17 p.

"As known, the deflections of the vertical can be computed by the formulae of Vening Meinesz. Although one has to know the gravity over the whole Earth, the influence of the nearest neighborhood of the computation point is the greatest. So, it is very important that a dense gravity survey be carried out around the point. However, this is not always possible, especially in mountainous areas. In that case, it is best to compute the influence of the nearest neighborhood from the topography.

In this paper is investigated the order of magnitude of the influence of nearby topography on the deflections of the vertical.

The influence is computed for several points for an area with a radius of about 5 km around the point. This shows that the influence of the nearest neighborhood can reach considerable amounts and may never be neglected in mountainous areas.

Further formulae are derived for the influence of the central zone on the deflections, assuming that the point lies on a slope. In the last chapter some models are studied in order to get a more general insight about the deflections caused by topography".

40 - STRANGE W.E. & G.P. WOOLLARD - Anomaly selection for deflection interpolation. Part I : Theoretical investigation. HIG-64-12. 91 p.
Part II : Practical application. HIG-64-13. 36 p.+ 14 t.
ACIC. USAF. Hawaii Inst. of Geophys. July 1964.

"Part I showed that :

a) The type of anomaly was not critical in computing deflection of the vertical if the Molodenskij theory was used and complete gravity knowledge existed,

b) What was critical was extrapolation of gravity values between observation points,

c) The complete Bouguer anomaly with geologic corrections provided the anomaly which allowed the most reliable extrapolations,

d) The normal complete Bouguer anomaly provided the most readily derived values for computation of deflections.

In Part II, the procedure outlined in Part I is applied to the problem of interpolation of deflections of the vertical.

In the test phase, three astro-geodetic deflection stations in the Rocky mountain area of the western United States were utilized. (Stations 102, 105 and 116 from USC & GS Special Publication n°229). Using the adopted procedure, the ξ deflection component was interpolated between stations 105 and 116 to obtain an interpolated ξ value at station 102. A comparison of this interpolated ξ value with the astro-geodetic ξ value showed that the two differed by only.22". Since the accuracy of the astro-geodetic values probably do not exceed $\pm$.2", the results were considered highly satisfactory.

In the application phase, deflection interpolation was carried out in an area in the Alps chosen by ACIC. In this case three deflection stations were chosen from a deflection of the vertical map provided by ACIC. Using the adopted procedure, the deflections components was interpolated between stations 1 and 3 to obtain a value at station 2. As there were no individual deflection values given, the astro-geodetic deflections were estimated from a η deflection component contour map with a 2.50" contour interval. Because of the manner of obtaining the astro-geodetic deflections, they are considered to have an accuracy of the order of $\pm$.5".

For this reason the difference between the computed and observed η deflection components, .59", has an uncertainty of $\pm$ 1.0". Since the difference again lies within the range of error inherent in the data the results are considered entirely satisfactory".

41 - ZABEK Z. - Problème de l'introduction du système des déviations gravimétriques de la verticale à la compensation du réseau astronomique et géodésique du pays.
Bull. Géod. 1961, n°61, p.213-244. (texte français)
Geodezja. Varsovie. 1961, n°6, p.3-47. (polish text, english abstract)

"The considerations concern an astronomical and geodetic network in which the gravimetric deflections of the vertical Θ (Σ) have been determined (or may be determined) on the Laplace points, with the assumption that the area of integration Σ is common for all points of the net and is limited by a spherical distance ψ_0, from the central point of investigated area...

... The author gives the relations between the components of the gravimetrically obtained deflections of the vertical and the geodetic elements of the net, where ξ (Σ') and η (Σ') denote the influence of the area Σ' omited at integration, and $\Delta\xi^{gr}$, $\Delta\eta^{gr}$ - the corrections of deflections of the plumb-line caused by the passage from the level ellipsoid to the reference ellipsoid of triangulation net".

ELLIPSOIDE de REFERENCE

VALEURS NORMALES de g

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21 - BALDINI A.A.* - New formulae useful when changing ellipsoidal parameters or orientation.
Gimrada. Fort Belvoir. 1962. Research note n°2, 16 p.

"This Report presents the derivation of new general equations for determining the changes in the deflections of the vertical and in the normal distance to an ellipsoid when changing the parameters or orientation of the ellipsoid. The equations are applicable in photogrammetric flash triangulation, Hiran, and other geodetic procedures involving datum ties. This work represents the progress to data in a research study the ultimate objective of which is development of mathematical procedures for determining the parameters of an ellipsoid the center of which is at the center of mass of the Earth".

22 - BILSKI E. - Tables of normal values of the gravity.
Trav. Inst. Polyt. Varsovie. 1964.
Geodezia, n°12, p.127-130 + 82 p. tables.

"The formulae of normal gravity used for computation of these tables are discussed. The origin of particular parameters of equations being examined, the problem of changing those formulae is brought up".

Tab. I : normal values at 0,01 mGal according to the Helmert formula (1901-1908) for the latitudes between 48 and 56° (interval : 0'1) p.4-35.

Tab. II : for the latitudes between 0 and 90° (interval:10') p.36-40.

Tab. III : normal values at 0,01 mGal according to the international formula, for the latitudes between 48 and 56° (interval : 10') p.41-72.

Tab.IV : for the latitudes between 0 and 90° (interval:10') p.73-77.

Tab. V : Differences at 0,01 mGal between the values of g from tab. IV and II, p.78-82.
(International Helmert).

* Ces articles font suite à ceux indiqués dans le Bulletin d'Information (Avril 1962) p.71.

23 - BURDJUKOV R.B. - Tables des valeurs normales de la pesanteur.
Gostop. 1961, 114 p.

Calcul des valeurs normales d'après la formule de Helmert, avec une précision de 0,01 mGal. Tables de corrections pour les autres formules (Cassinis, Krassovsky ou Zongolovic).

24 - BURSA M. - Bestimmung der Dimensionen des Erdellipsoids Auf Grund der Anomalen Krümmungen des Quasigeoids.
Studia Geophys. & Geod. 1963, v.7, n°2, p.108-117.
(texte russe, résumé allemand)

"In der vorliegenden Abhandlung wird ein Versuch der Ableitung der Dimensionen des Erdellipsoids auf Grund der Krümmungsmasses und der mittleren Krümmung vorgenommen. Es werden die Forderungen $\sum \delta K^2$ = Min. und $\sum \delta J^2$ = Min. formuliert, wobei mit δK der Unterschied des Krümmungsmasses und δJ der Unterschied der mittleren Krümmung des Quasigeoids und des zu bestimmenden Erdellipsoids bezeichnet werden. Der Ableitung der Theorie lagen die Arbeiten von Molodenskij sowie deren Weiterbearbeitung von Eremeev und Jurkina zugrunde, wo die Hauptkrümmungshalbmesser des Quasigeoids bei der Herleitung des mittleren Erdellipsoids, gemeinsam mit den gravimetrischen Angaben, angewendet worden sind. Diese Theorie ist vorteilhaft, da sie keine geodätische Verbindung der für die Parameterableitung des Erdellipsoids benutzten Netze erfordert. Als Nachteil kannjedoch die Unmöglichkeit einer direkten Ableitung der Orientationsglieder des neuen Erdellipsoids bezeichnet werden.

Diese Theorie wurde auf die unverbundenen geodätischen Systeme Europas und Kanadas appliziert, die auf die Erdellipsoide von Hayford und Clarke (1866) bezogen sind. Auf Grund der Forderung, dass die Summenwerte der Quadrate der anomalen Krümmungen des Quasigeoids dieser Gebiete minimal sein sollen, wurde - bei der erwählten Abplattung 1 : 298,3 - der Wert der halben Hauptachse des Erdellipsoids für Europa und Kanada mit a = 6378167 m abgeleitet. Die mittleren Anomalienwerte des Krümmungsmasses und der mittleren Krümmung des Quasigeoids für 18 Teilgebiete Europas und Kanadas wurden aus den abgeleiteten Parametern der mittels der bekannten projektiven Methode bestimmten bestanschliessenden Erdellipsoide errechnet.

Die anomalen Krümmungen δK, δJ (der Linien δK = Konst., δJ = Konst) können auch als Charakteristiken des ausgeglätteten Quasigeoids in den untersuchten Gebieten angewendet werden".

25 - BURSA M. - Determination of the curvature of the surface representing the figure of the Earth from astrogeodetic and gravimetric data.
Research Inst. Geod. Prague. 1963, 4 typewritten pages.

..."The advantage of this method for determination of the dimensions of the Earth's ellipsoid consists in the fact that the exact geodetic connection of the used geodetic systems is not necessary.

A disadvantage of this method is the impossibility of determining the geodetic datum. By this way only values of the semi-major axis and of the flattening can be obtained".

26 - BURSA M. - Détermination des paramètres de l'ellispoide de référence convenant aux réseaux géodésiques européens d'après les données astronomico-géodésiques du catalogue de l'A.I.G.
Bull. Géod. 1963, n°68, p.139-144.

"A l'Institut Géodésique de recherches à Prague, nous nous sommes occupés de la solution de ce problème dont nous communiquons ici certains résultats et conclusions...

Par rapport au caractère hétérogène des données astronomico-géodésiques employées, nous avons choisi pour la détermination des paramètres de l'ellipsoide la méthode des aires connue dans ses 2 variantes : la méthode translative et la méthode projective...

En particulier, les résultats ont montré "que les dimensions de l'ellipsoide le plus convenable pour l'Europe sont en comparaison avec celles des ellipsoides connus, introduits en pratique géodésique, le plus proche des dimensions de l'ellipsoide de Krassowskij"... Ils ont confirmé de plus "que les dimensions de l'ellipsoide de Hayford sont trop grandes en comparaison avec celles de l'ellipsoide le plus convenable pour l'Europe..."

27 - BURSA M. - Corrections to the European Datum by means of the European astrogeodetic Network. (shorter contribution)
Studia Geophys. & Geod. 1963, v.7, n°2, p.284-85.

..."The astrogeodetic deflections were used at the Research Inst. of Geod. in Prague for solving the problem of the Reference Ellipsoid for Europe (a = 6 378 112 m, α = 1/298.4).

The computation of the corrections $d\xi_o$, $d\eta_o$ on the basis of the above materials is given..."

28 - BURSA M. - On the determination of the direction of the minor axis of the reference ellipsoide and the plane of the initial geodetic meridian from observations of artificial Earth satellites.
Studia Geophys. & Geod. 1965, v.9, n°2, p.16-21.
(russian text, english abstract)

"In this paper, following upon the problem of determining the directions of the basis axes of the geodetic reference system with respect to the astronomical system (X, Y, Z,) directionally defined by the direction of the Earth's axis of rotation and the plane of the initial astronomical (Greenwich) meridian, is solved...

In the conclusion of the paper, it is shown how the results of simultaneous observations of satellites can be used for azimuth control of astrogeodetic network and also for determining respective corrections of geodetic coordinates and quasigeoidal heights, if one of the two satellite stations, from which the position of the satellite is simultaneously determined, is fixed. As an example, Finnish observations carried out at points Turku and Helsinki are used, together with equations containing Euler's angles ε , ψ and ω for Hayford's ellipsoid with the Potsdam orientation according to the "European Datum". For the exact determination of ε, ψ and ω , it is, of course, necessary to have a larger number of pairs of astrogeodetic points with simultaneous satellite observations. If the used measurements do not contain considerable systematic errors, i.e. if the absolute terms in Eq. (28) are real, Euler's angles for the axes of the Hayford ellipsoid can be as much as several seconds in magnitude".

29 - BURSA M. - Theory of determination of position of centre of reference ellipsoid from observations of Earth satellites.
Studia Geophys. & Geod. 1965, v.9, n°3, p.225-229.

"The position of the centre of a reference ellipsoid with respect to the centre of mass of the Earth ΔX_o , ΔY_o, ΔZ_o, can be calculated but of course, the exact values of all elements of the satellite orbit for the moment of observation must be known. If only 2 elements, defining the position of the orbital plane, i.e Ω and i, are known, then the problem can be solved from the equation of the osculation orbital plane in the geocentric system X, Y, Z. The coordinates ΔX_o, ΔY_o, ΔZ_o must then satisfy condition where, however, the Euler angles ε, ψ , ω , defining the direction of the geodetic reference system X', Y' Z' with respect to the system X, Y, Z, are unknown. It seems to be better to determine them from simultaneous measurements of the directions to the satellites from given geodetical satellite stations. It is then possible to use the equations of corrections in the form where the influence of ε , ψ and ω is included in the absolute term and only the sought quantities, ΔX_o , ΔY_o and ΔZ_o, are unknown. A pre-condition for this, of course, is that the used values of the elements Ω and i define a really geocentric orbit. This assumption need not always be satisfied, as is shown by Eqs..."

30 - HEISKANEN W.A. - Is the Earth a triaxial ellipsoid ?
Geophys. Research. J. 1962, v.67, n°1, p.321-327.

31 - JEFFREYS H. - On the expression of gravity formulae.
Geophys. J. 1964, v.8, n°5, p.541.

"Recent studies of the Earth's field have expressed all the terms depending on the figure in terms of J_2. I think it would be better to retain the ellipticity e as a factor...
The difference between geocentric and geographic coordinates contains e as a factor in any case".

32 - MIKHAILOV A. - On the new system of astronomical constants.
Studia Geophys. & Geod. 1965, v.9, n°2, p.176.

List of the astronomical constants (of geodetic interest) defined at the General Meeting of the I.A.U. Hamburg 1964.

33 - RAPP R.H. - A consideration of Hayford's best fitting ellipsoid data using the differential change equation of Vening Meinesz.
Geof. Pura Appl. Milan. 1963, v.54, p.1-5.

"The relations of the defining parameters of a geodetic system to the deflections of the vertical in this system given by Vening Meinesz have been used in the computation of a best fitting ellipsoid. The data was that used by Hayford in the computation of the resulting International Ellipsoid. Assuming the undulation (N_0) at Meades Ranch to be 0, the results are : a = 6,378,194 m and I/f = 299 9. This a is much nearer to modern values than that of the International Ellipsoid. If a flattening of 1/298 24 is fixed and N_0 = 3 m, a = 6378164 m which differs by one meter from a recent determination given by Kaula".

34 - UOTILA U.A. - Harmonic analysis of world wide gravity material.
Publ. Isos. Inst. 1962, n°39, 17 p.
(Ann. Acad. Sci. Fenn. Ser. A, III, Geol. Geogr. n°60)

(This publication is the same as the next one (n°35) but, with not so many mathematic expressions).

"Dr. Uotila has computed corrections to the International formula and formula for the triaxial ellipsoid. He computed also the lower degree spherical harmonics, using surface gravity anomalies above, and computing surface gravity information and zonal harmonic obtained from satellite data.

He carried out the computations using the mean free air anomalies of 11,586 squares of 1° and the isostatic anomalies of 8,172 squares of 1°. This study indicates that the northern and southern hemisphere give different gravity formulae. Whether this discrepency is caused by the different distribution of the oceans on both hemispheres or by the too scarce gravity observations on the southern hemisphere, has to be studied. The following results for the whole world was obtained :

$$\gamma_o = 978.0404\ (1+0.005\ 3014\ \sin^2\varphi - 0.0000059\ \sin^2 2\varphi)\text{Gal},$$

corresponding flattening is 1/298.15.

35 - UOTILA U.A. - Corrections to gravity formula from direct observations and anomalies expressed in lower degree spherical harmonics.
I.G.P.C. Columbus. 1962, Rep. n°23, 26 p.

See page 65.

36 - UOTILA U.A. - Coefficients of low degrees spherical harmonics of gravity field as computed from surface gravity anomalies.
from : "The use of artificial satellites for geodesy".
(Washington D.C. 1963. April 26-28), p.353-357.

"An analysis in spherical harmonics of the Earth's gravity field has been made using free air anomalies and isostatic anomalies. Terms up to the fourth order have been determined. A second solution has been made by keeping fixed the values of the zonal harmonics obtained from satellite data".

37 - VEIS G. - The position of the Baker-Nunn camera stations.
Smith. Inst. Astr. Obs. Cambridge, Mass. 1961, Sp. Rep. n°59, 5 p.

"An attempt has been made to reduce the coordinates of the Baker-Nunn camera stations to a uniform geodetic system based on international ellipsoid.

We did not use the classical method suggested by Helmert or that of Vening Meinesz (1950) which employ the elliptical coordinates. Instead, we developed a new method using regular coordinates. This method is much simpler and is valid for any distance between the two systems to be connected..."

38 - WOLF H. - Die heutigen numerischen Grundlagen zur Reduktion von Triangulationsmessungen auf das Ellipsoid.
Texte ronéotypé présenté à la 12 ème A.G. de l'U.G.G.I.
Helsinki, 1960.
Bull. Géod. 1963, n°68, p.149-163.

39 - WOLF H. Dreidimensionale Geodäsie Herkunft, Methodik und Zielsetzung.
Z. VermessWes. 1963, t. 88, H.3, S.109-116.

40 - WOLF H. - Die Grundgleichungen der Dreidimensionalen Geodäsie in elementarer Darstellung.
Z. VermessWes. 1963, t.88, H.6, S.225-233.

41 - ZABEK Z. - Reduction of astronomical observations on the geoid and geodetic observations on the reference ellipsoid.
Trav. Inst. Polyt. Varsovie. 1964.
Geodezja n°12, p.33-46.
(polish text, russian and english abstracts)

"After having discussed the Helmert's projection method which consists in projecting the points of Earth's surface along the normals to the reference ellipsoid and the Pizetti's method where points of the Earth's surface are firstly transferred on the geoid along vertical lines and then projected upon the reference ellipsoid, more preference is given to this last method by the author. To apply this method for elaboration of astro-geodetic network, a reduction of astronomical observations on the geoid due to the curvature of vertical line is to be performed as well as the reduction of geodetic observations on the reference ellipsoid due to the elevation of points of observation above the ellipsoid and the deviations of the vertical. Reduction of astronomical observations is discussed in dependence on the system either of orthometric heights or normal heights of Molodenskij, as adopted for elaboration of geometrical levelling. The influence of curvature of the vertical upon the geodetic elements of the net being negligeably small, the formulae as derived for the reduction of angles and distances do not depend on the height system adopted".