Growth and decay of a convective boundary layer over a surface with a constant temperature

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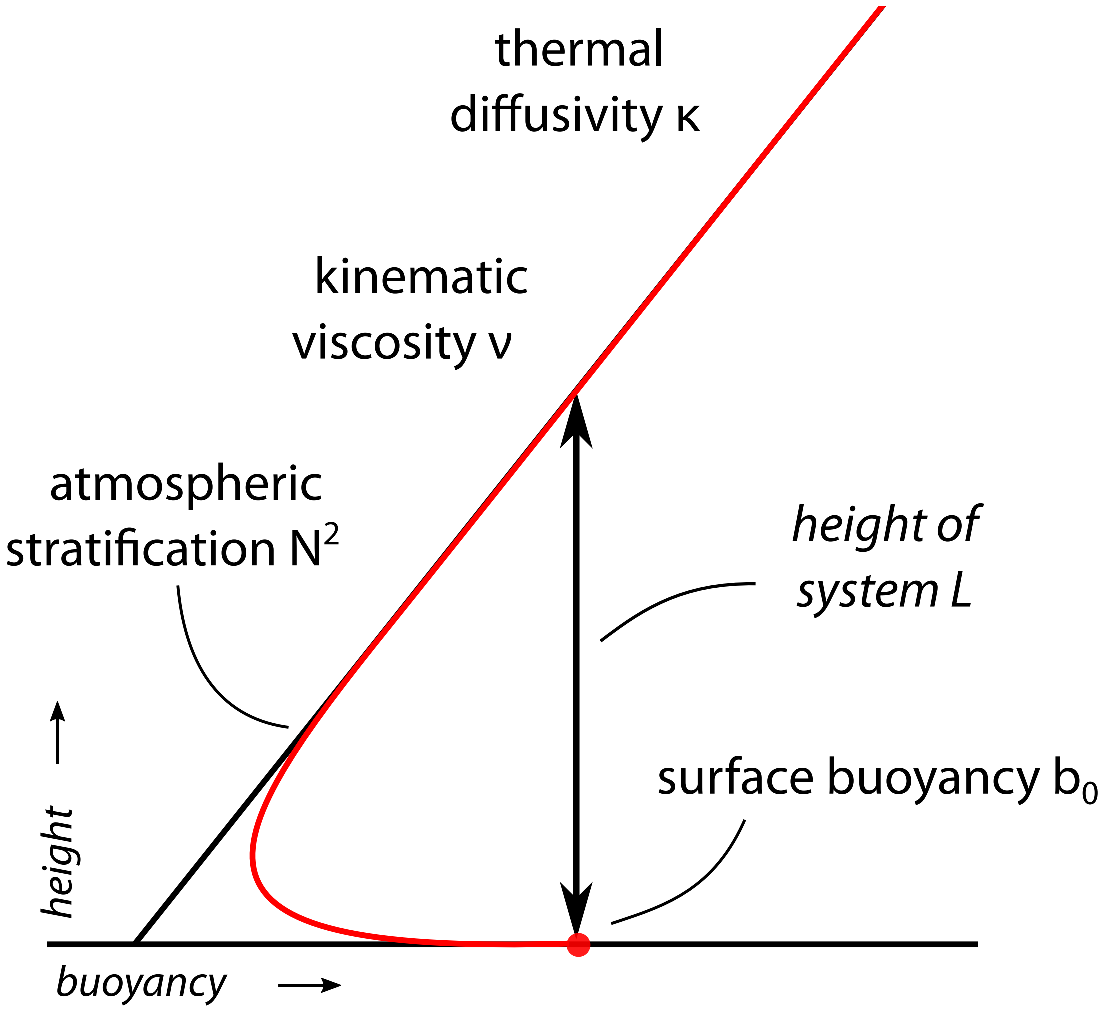
A canonical case of atmospheric turbulence

- A very simple configuration
  - Linear stratification
  - A constant surface temperature
  - Buoyancy as thermodynamic variable

$$b \equiv \frac{g}{\theta_{v0}} \left(\theta_v - \theta_{v0} \right)$$

  - In total four parameters

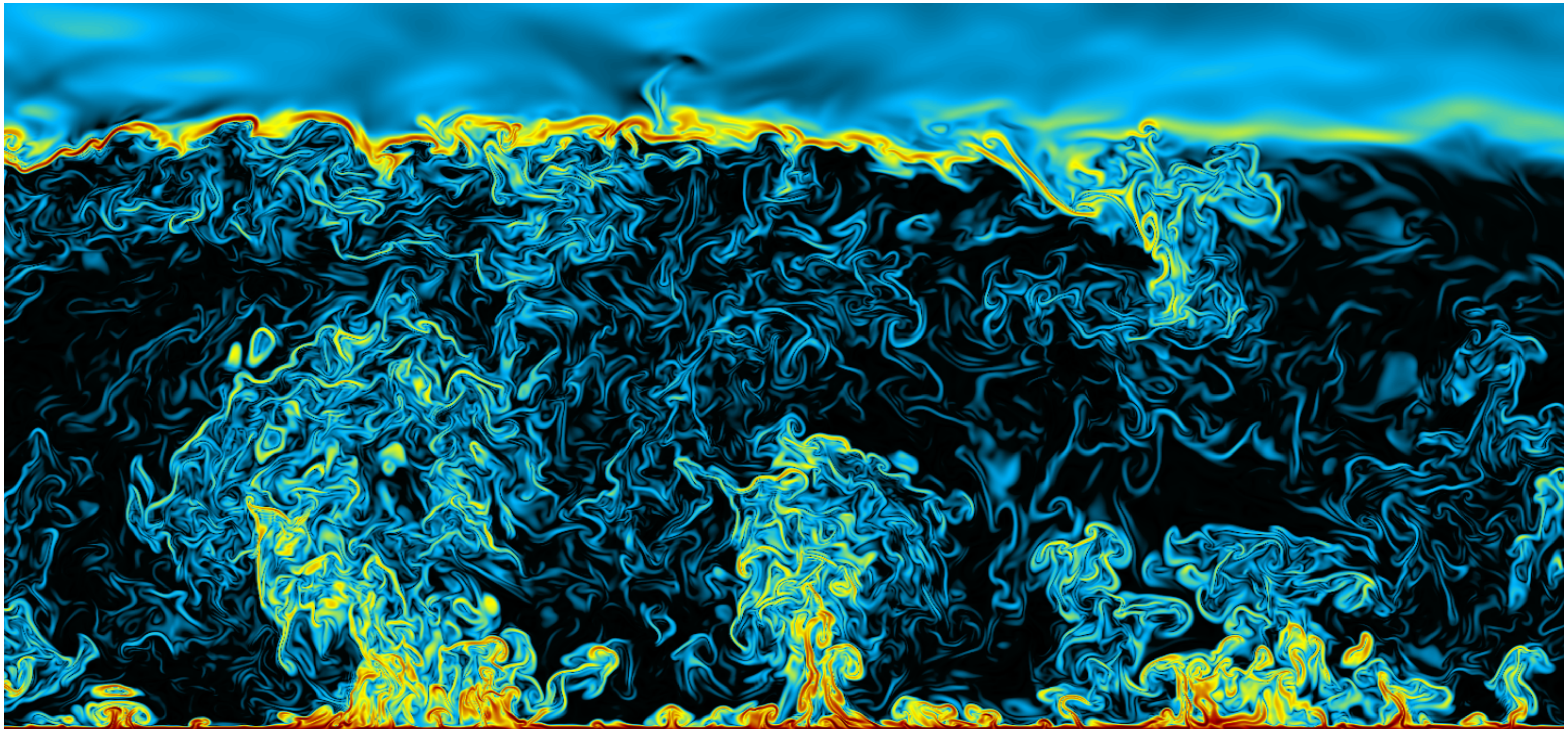


- A detailed description of this reference case is lacking in literature

- Why do we want to study this?
  - Decay of turbulence: the afternoon transition

How does the system evolve in time?

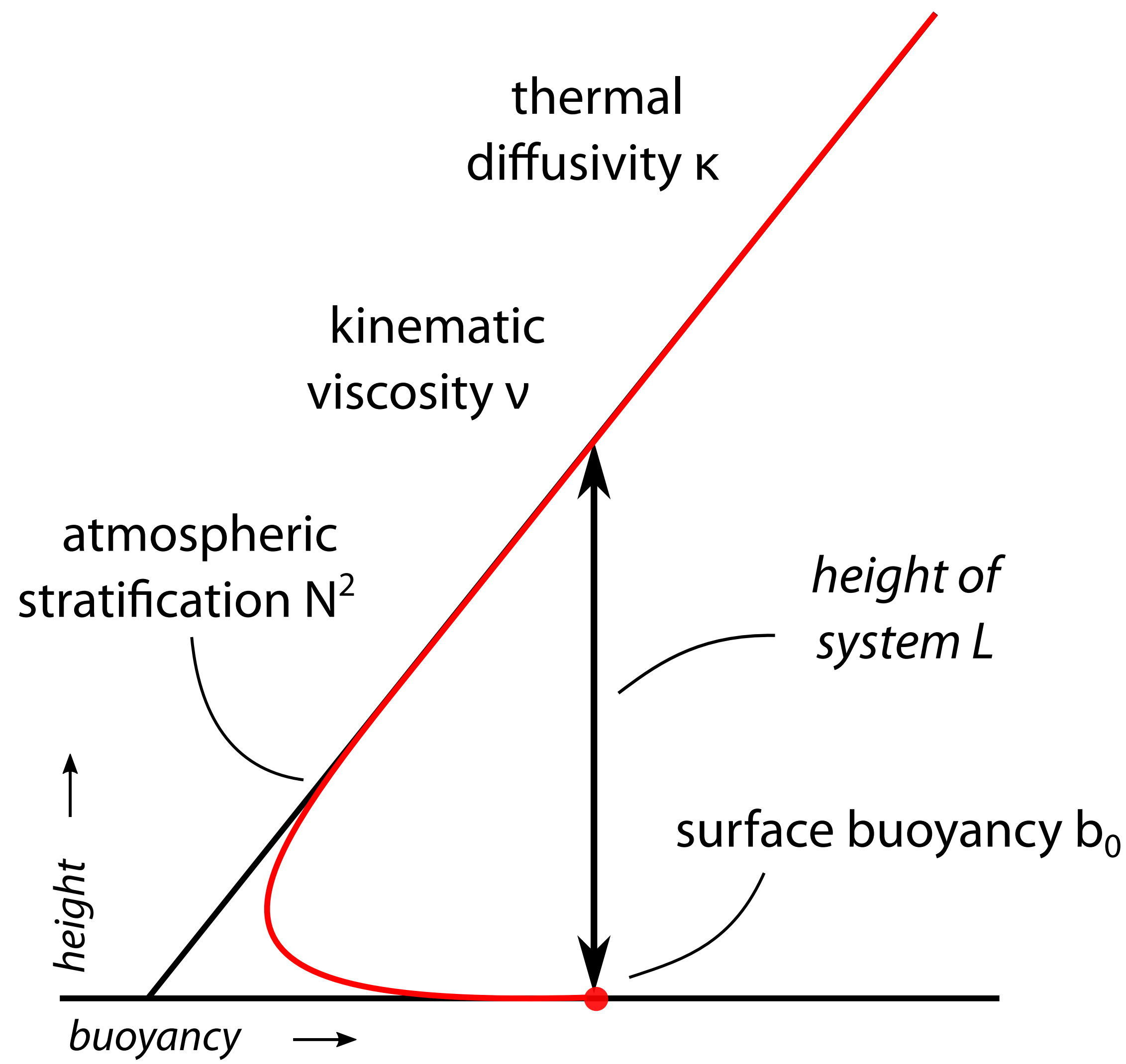
- Surface flux reduces in time towards zero
- Height increases in time towards maximum height *L*
- Vertically integrated kinetic energy reaches peak and then decays



Magnitude of the temperature gradient vector

Derivation of a non-dimensional system and its two parameters

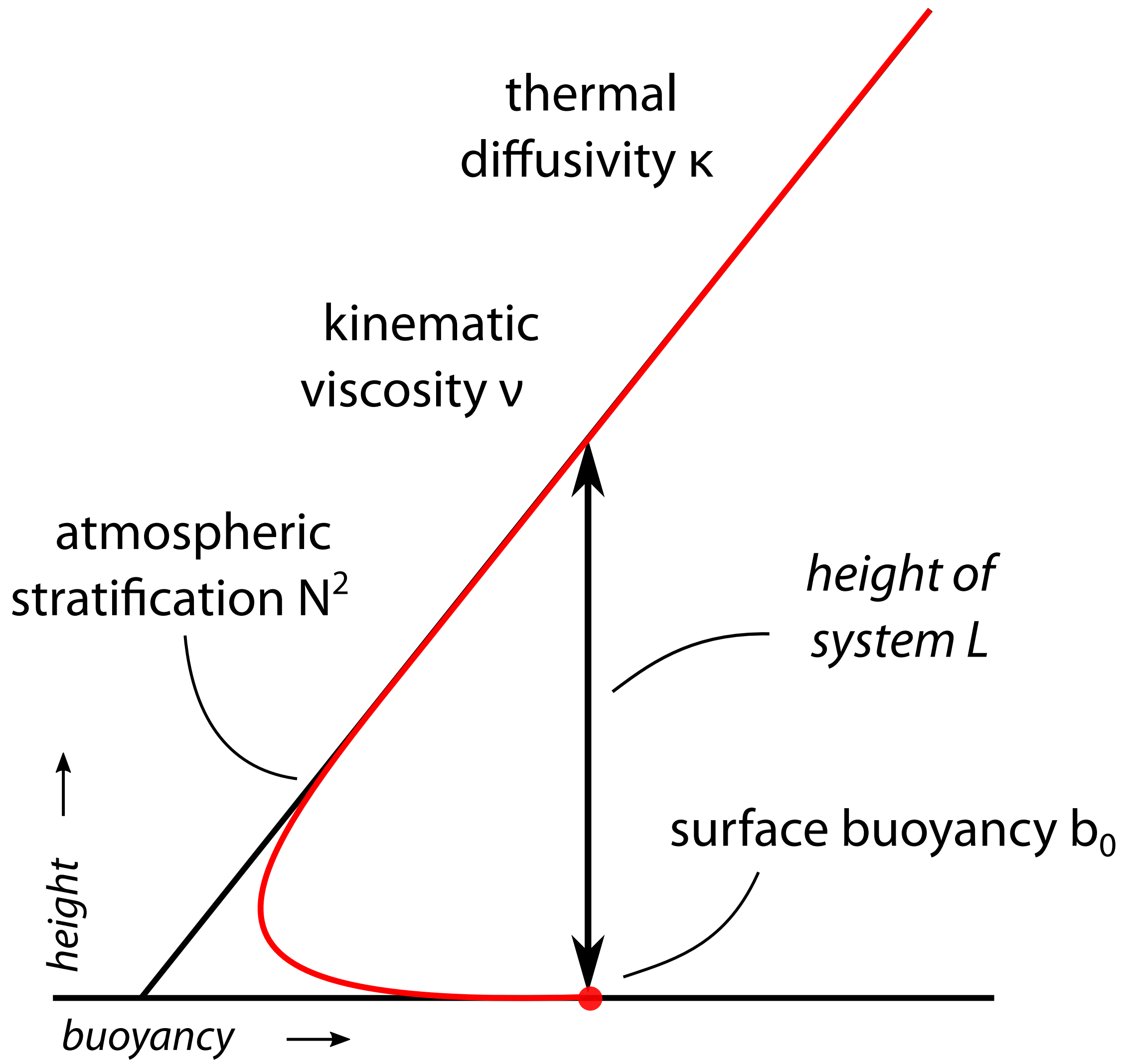
- Outer length scale $L \equiv \frac{b_0}{N^2}$
  largest length scale
- Buoyancy flux scale $B \equiv b_0^{\frac{4}{3}} \kappa^{\frac{1}{3}}$
  viscous scaling
- Velocity scale $U \equiv (BL)^{\frac{1}{3}} = \frac{b_0^{\frac{7}{9}} \kappa^{\frac{1}{9}}}{N^{\frac{2}{3}}}$
  convective scale
- Time scale $T \equiv \frac{b_0 L}{B} = \frac{b_0^{\frac{2}{3}}}{N^2 \kappa^{\frac{1}{3}}}$
  time to warm reservoir (volume / flux)



$$\left\{ Pr \equiv \frac{\nu}{\kappa}, Re \equiv \frac{UL}{\nu} \right\}$$

Direct numerical simulations of four Reynolds numbers

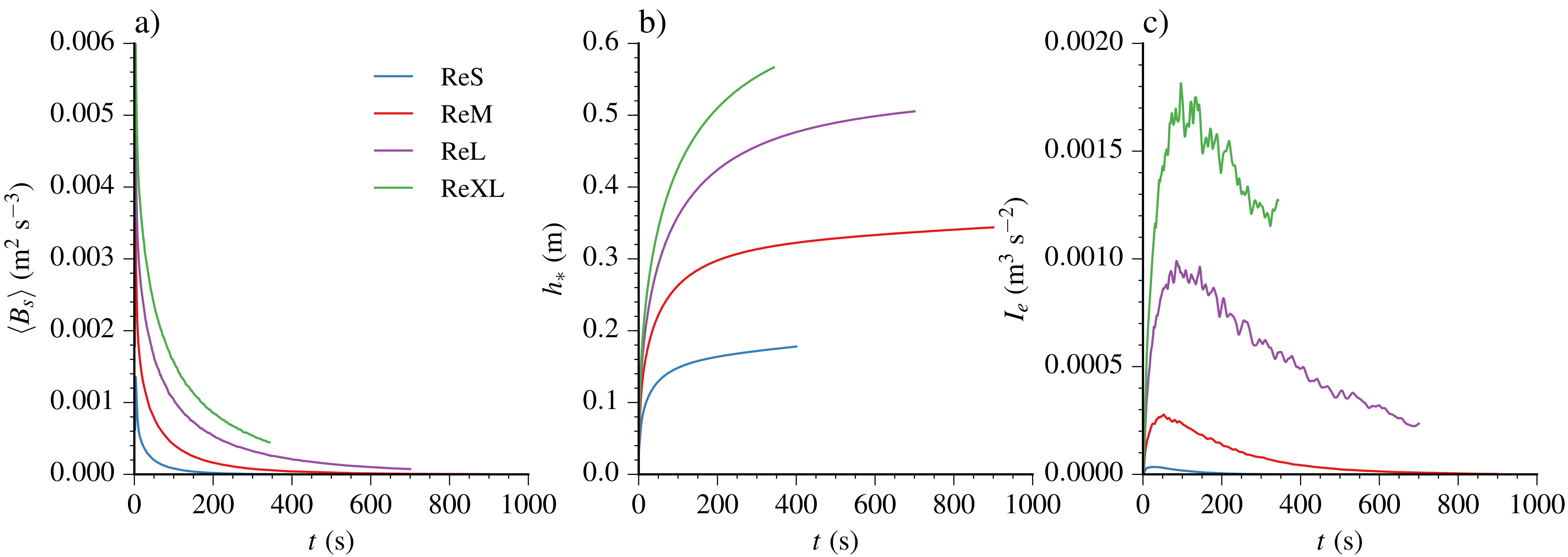
- Direct numerical simulation
- Prandtl number of unity
- Four different Reynolds numbers
- Horizontal dimensions domain 2 x 2 m
- Domain height dependent on *L*



Name	$N_x \times N_y \times N_z$	b_0	b_0/N^2	ν, κ	Re
ReS	$1024 \times 1024 \times 384$	0.5	0.1667	1×10^{-5}	285
ReM	$1024 \times 1024 \times 768$	1.0	0.3333	1×10^{-5}	718
ReL	$1536 \times 1536 \times 768$	1.6	0.5333	5×10^{-6}	2133
ReXL	$2048 \times 2048 \times 1024$	2.0	0.6667	5×10^{-6}	2873

Time evolution of three relevant variables in all simulations

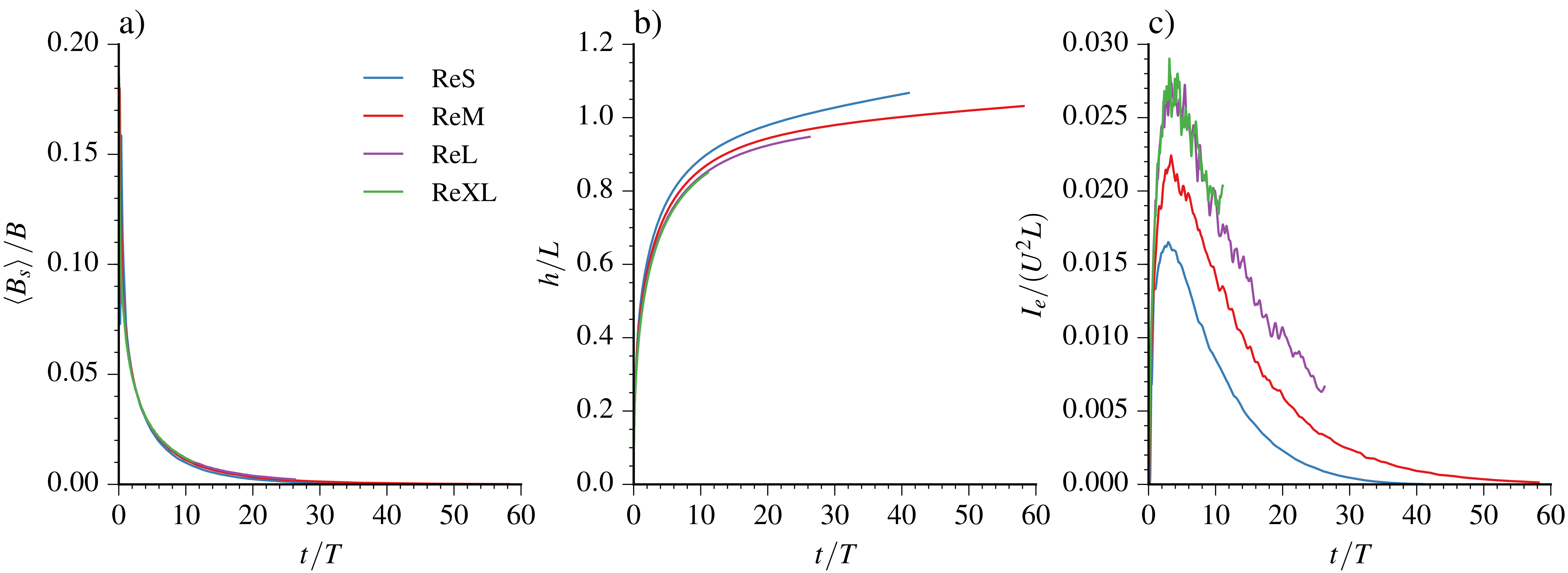
- Surface buoyancy flux decays towards zero
- Boundary layer depth evolves towards maximum given by *L*
- Integrated kinetic energy peaks early and then slowly decays



surface buoyancy flux *boundary layer depth* *integrated kinetic energy*

Non-dimensional results

- With scaling parameters *L*, *T* and *B* results can be plotted non-dimensional
- Non-dimensionalization leads to collapsing graphs
- Results of *ReL* and *ReXL* converge: extrapolation to atmosphere possible

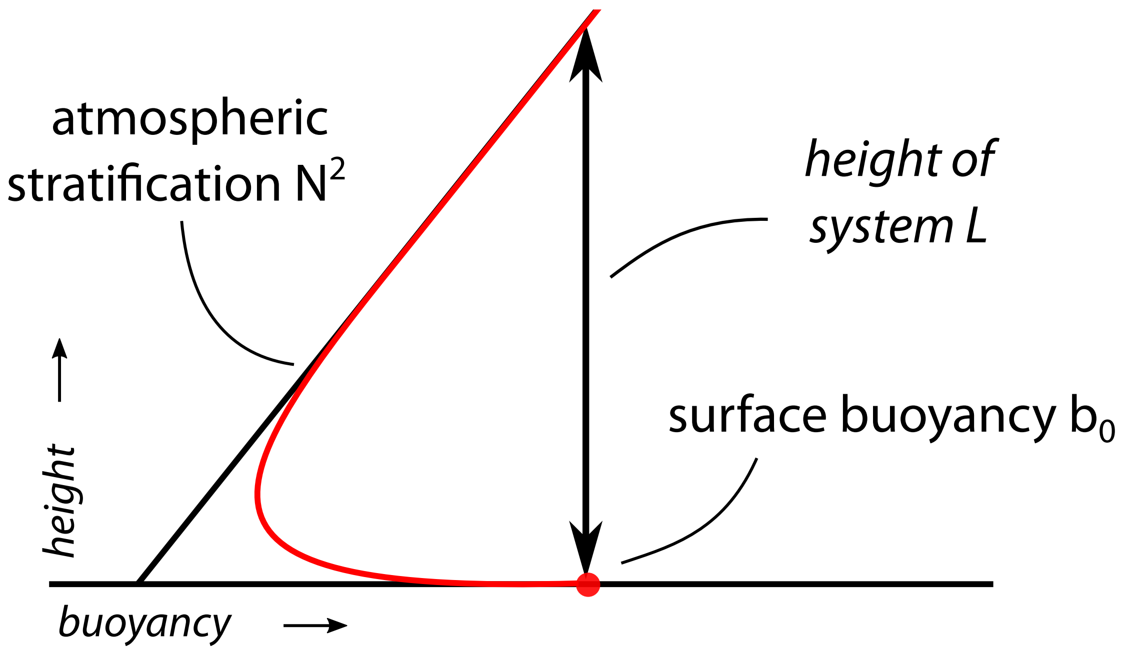


surface buoyancy flux *boundary layer depth* *integrated kinetic energy*

Derivation of a mathematical model

- Our chosen boundary layer height is an integral length scale

$$h_*^2 \equiv \frac{2}{N^2} \int_0^\infty \langle b \rangle - N^2 z \, dz$$



- We can derive an exact expression for the rate of change of this length scale based on the simulation

$$\frac{dh_*^2}{dt} = \frac{2}{N^2} \left(B_s + \kappa N^2 \right)$$

Non-dimensional form of the mathematical model

- Differential equation is the same as that for a bulk model without entrainment (encroachment model)

$$\frac{dh_*}{dt} = \frac{B_s + \kappa N^2}{h_* N^2}$$

- Equation can be rewritten in non-dimensional variables with the help of the defined length, time and buoyancy scales *L*, *T* and *B*

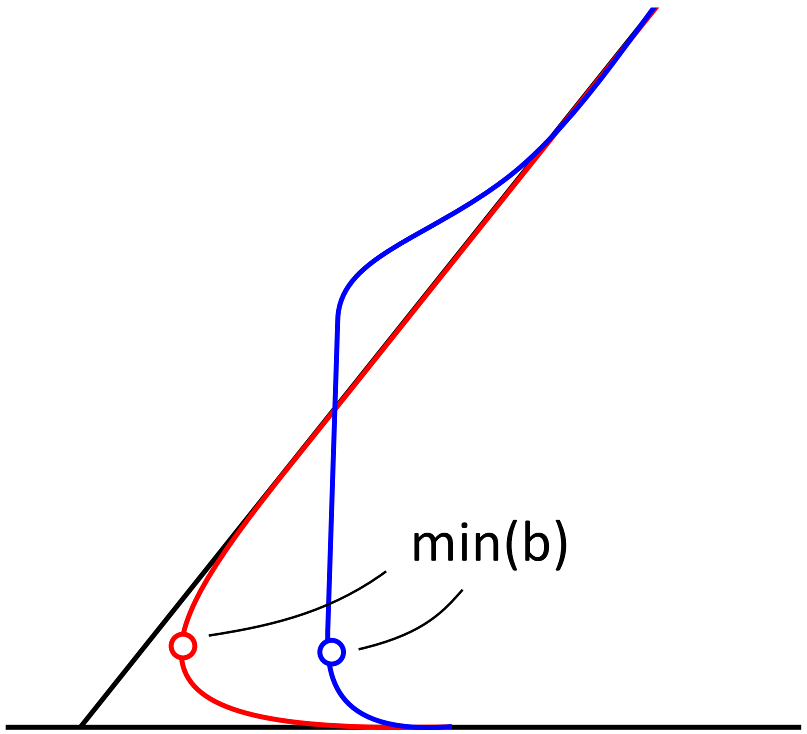
$$\frac{d\widehat{h}_*}{dt} = \frac{\widehat{B}_s + Re^{-\frac{3}{4}}}{\widehat{h}_*}$$

- For high Reynolds numbers, second term in numerator can be neglected

$$\frac{d\widehat{h}_*}{d\widehat{t}} = \frac{\widehat{B}_s}{\widehat{h}_*}$$

A model for the surface buoyancy flux is needed

- We propose a model for the surface buoyancy flux
  - The mixed-layer value is approximated by min(b)



$$
\begin{aligned}
B_s &\equiv c_0 \kappa^{\frac{1}{3}} \left(b_0 - \min(b)\right)^{\frac{4}{3}} \\
&= c_0 \kappa^{\frac{1}{3}} \left(b_0 - c_1 h_* N^2\right)^{\frac{4}{3}}
\end{aligned}
$$

- The values for the constants can be derived from the simulations

Final form of the mathematical model for our system

- Substitution of the analytical model for the surface buoyancy flux gives

$$\frac{dh_*}{dt} = \frac{c_0 \kappa^{\frac{1}{3}} \left(b_0 - c_1 h_* N^2\right)^{\frac{4}{3}}}{h_* N^2}$$

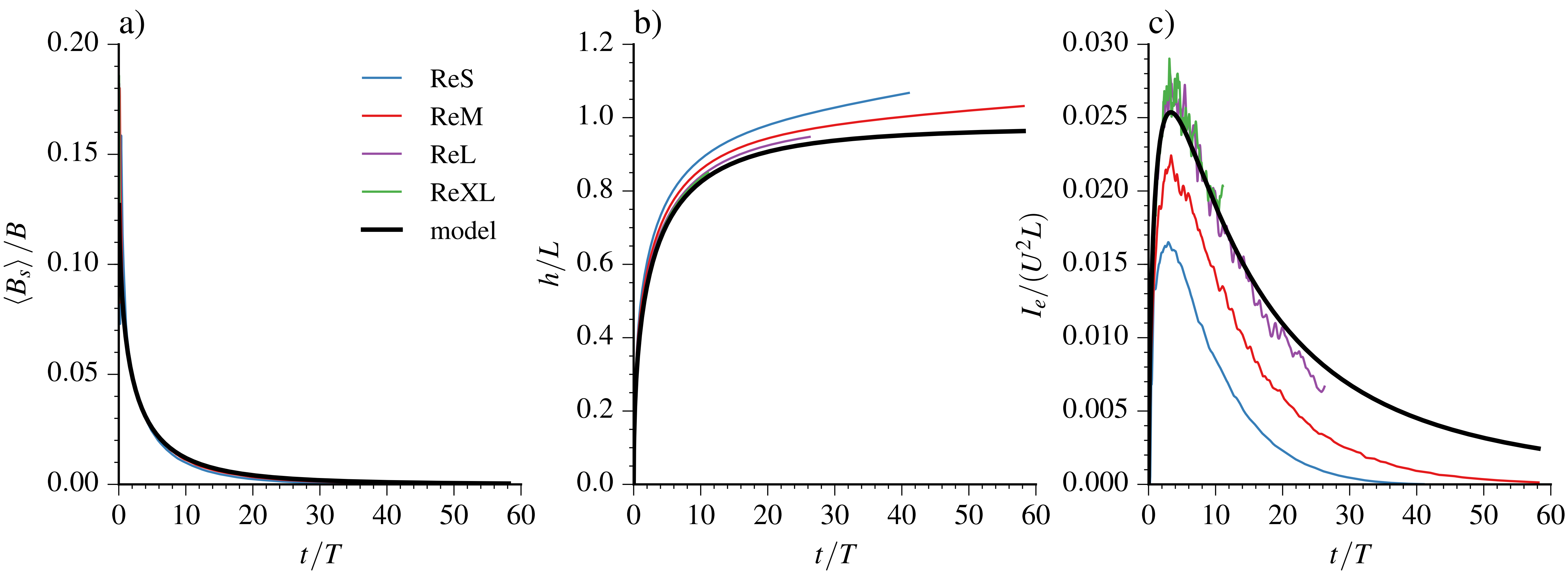
- In terms of non-dimensional variables this is

$$\frac{d\widehat{h}_*}{d\widehat{t}} = \frac{c_0 \left(1 - c_1 \widehat{h}_*\right)^{\frac{4}{3}}}{\widehat{h}_*}$$

- This can be solved analytically and all the scaling variables can be derived

Validation of the derived mathematical model

- The model matches very well with the data
- Nearly perfect match for the two highest Reynolds number cases
- Model predicts decay rate of kinetic energy in a system that slowly dies out

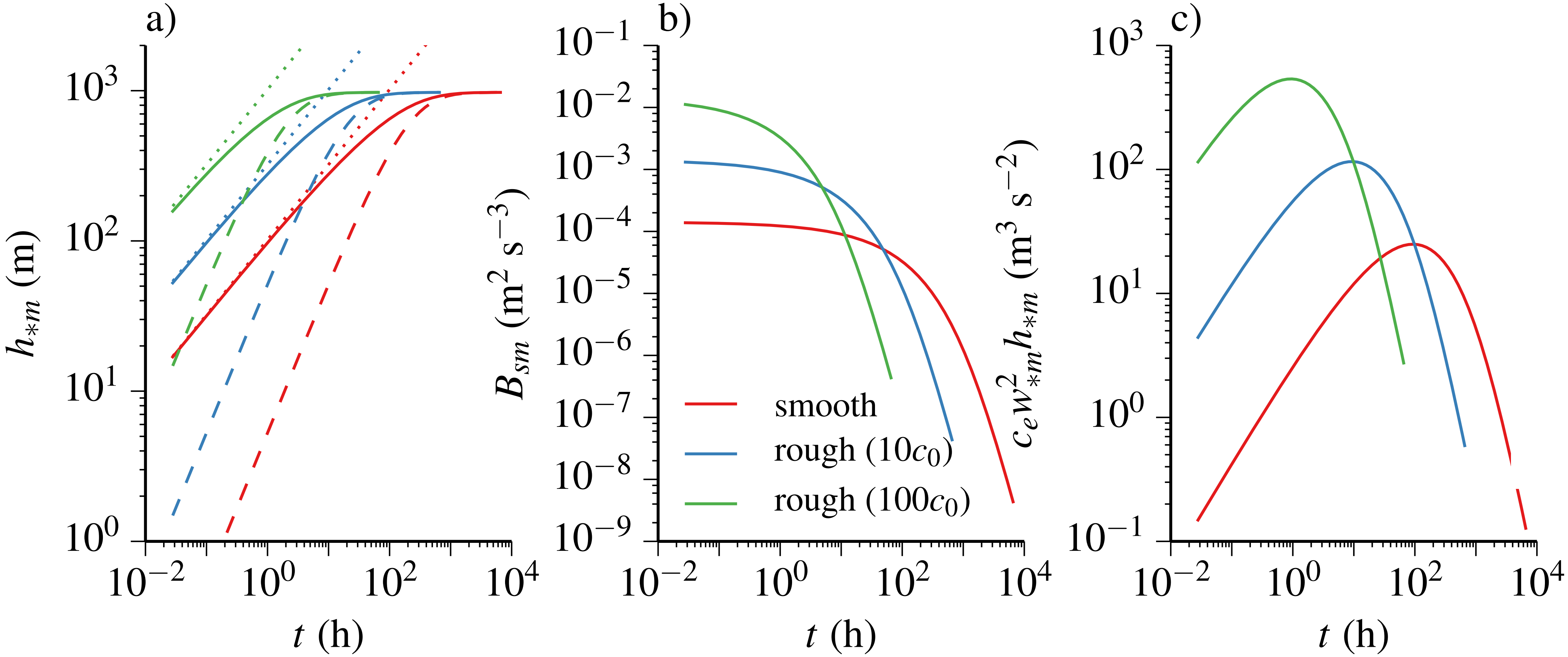


surface buoyancy flux *boundary layer depth* *integrated kinetic energy*

Back to typical atmospheric dimensions

- Typical atmospheric conditions
  - Excess surface temperature 6 K, lapse rate 6 K km^{-1}, thus L = 1000 m
  - Three exchange rates: smooth, moderately rough and very rough surface

- Decay does not follow a power law, but has increasingly negative slope



boundary layer depth *surface buoyancy flux* *integrated kinetic energy*

Conclusions

- The convective boundary layer over surface with a fixed surface temperature
  - Complex transient system with a peak in integrated kinetic energy
- Direct numeral simulations of system can be extrapolated to atmosphere
  - Reynolds number similarity presented
- A model derived for high Reynolds number flows
  - Mathematical model is able to predict bulk characteristics of the system
  - Model can be used to predict the afternoon transition of the CBL
- The decay of kinetic energy in the boundary layer does not follow power law

van Heerwaarden and Mellado, 2016. Growth and decay of a convective boundary layer over a surface with a constant temperature. *Journal of the Atmospheric Sciences*. Under review.